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DISTRIBUTED MODEL PREDICTIVE CONTROL FOR MULTI-OBJECTIVE WATER SYSTEM MANAGEMENT

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Abstract

Model Predictive Control (MPC) has been implemented on large-scale water systems in the Netherlands with the objective to keep water levels within a certain range. However, the application of conventional centralized MPC is not possible for water systems that are very large, especially when multiple organizations with their own objectives are involved. Distributed Model Predictive Control is introduced in order to deal with multiple goals in a computationally tractable way. In this paper, we illustrate how dual decomposition could be used for establishing the multi-objective management in a distributed manner.

Keywords: Model predictive control; Distributed control; Multi-objective management; Water system management

1. INTRODUCTION

Water is one of the most important elements in the world. It is used everyday, for drinking, agriculture, navigation, energy production, etc. In the Netherlands, the water systems consist of rivers, lakes and estuaries, which are connected by natural rivers and artificial canals. Some rivers originate from neighboring countries, whose discharges vary over different seasons. Reservoirs, lakes and canals in the western and northern parts of the country are controlled by gates and pumps in order to prevent mixture of the fresh water with the sea water, especially during periods of severe storms, which may bring the water level up to 4 m above the sea level [1]. Water managers continuously manipulate the water levels to meet several requirements [2]:

(a) Safety—in the case of inundation, water levels have to be controlled below the maximum level allowed.
(b) Water demand—to make sure that enough drinking water and other daily consumption is available, water levels have to be controlled above the minimum level allowed.
(c) Navigation—for ships sailing through the rivers, water levels have to be maintained around a certain point or reference level.

Structures like pumps and gates are used to manipulate the flow in the rivers and water courses, which always have limited capacities. In general, the management of this kind of water system is not simple, as the objectives (a)-(c) may be conflicting with one another. For example, protections from inundation during high flow periods and guarantees for enough drinking water have higher priority than other situations. Navigation and fresh water for agriculture during regular periods also have to be ensured in the management. Many methods have been used in the water management over the last decades, such as feedforward control [3], feedback control [4], and Model Predictive Control (MPC) [1]. In general, feedback controllers show a delay in their actuation and feedforward controllers combined with feedback controllers work well until the required control flow exceeds the maximum pump capacity [2]. Finally, MPC is the control technique that shows the best performance for this kind of problems. MPC is a state-of-the-art approach to control water levels to fulfill multiple goals and deal with the delay time that exists in real water systems. Besides that, constraints and uncertainties can be considered explicitly into MPC as well. As a result, MPC has the potential to perform better than the other two methods and has become a popular control scheme due to its versatility [5]. However there are still two problems that arise when MPC is implemented on large-scale systems [6]: (a) for large-scale water systems, different water boards take charge of different aspects of water management separately. For example water quantity and water quality are not always managed by the same authority. And sometimes these authorities have limited communication with each other which means they only manage the water in a local manner. But in centralized MPC, the controller manipulates all aspects overall; (b) even if those issues can be solved in an integrated manner, it may take so much computational time for a centralized MPC to calculate a large-scale water system problem. For example, solving the large water systems in the Netherlands is not tractable, due to computational limitations.

In order to improve the application of MPC on large-scale systems and relieve the computational burden as much as possible, we use Distributed Model Predictive Control (DMPC) algorithms. Distributed control uses communication among the local controllers to reduce the side effects that their decisions have on neighboring subsystems. In this way, local controllers or agents are able to reach an agreement about what control action should be applied, minimizing the loss of control performance with respect to an ideal centralized MPC controller that would control the overall system [1, 6, 7]. In this case, the heavy computational burden in a large scale system can be relieved since each subsystem is controlled by a local agent separately, which works only with a reduced model and partial information [6, 8, 9].

In this paper, we show that DMPC can be used in order to separate a multi-objective optimization into several components related with the respective objectives which have to be minimized. As a result, it is possible that different parties or entities participate in the optimization in order to guarantee their particular objectives.

The remainder of this paper is organized as follows. In Section 2, a model of a low-land water system is introduced and multi-objective control problem is formulated. In Section 3, centralized and distributed MPC are introduced and utilized to solve the control problem. In Section 3, simulation results are presented and Section 4 concludes the paper and contains directions for future work.

2. PROBLEM FORMULATION

In this paper, we will apply DMPC on a low-land reservoir system, which is shown in Figure 1. Firstly, in this water system, water from precipitation will flow into the system through upstream rivers and canals as an inflow Q_d (m^3/s). Secondly, water is pumped out of the system as an outflow Q_c (m^3/s). Finally, the water level in the system, h (m), is the variable that we need to regulate by adjusting Q_c. Note that Q_d is a stochastic variable due to the chaotic nature of weather, Q_c has to be chosen taking into account maximum and minimum pump capacity constrains and h has to be maintained within a range as depicted in Fig. 1.

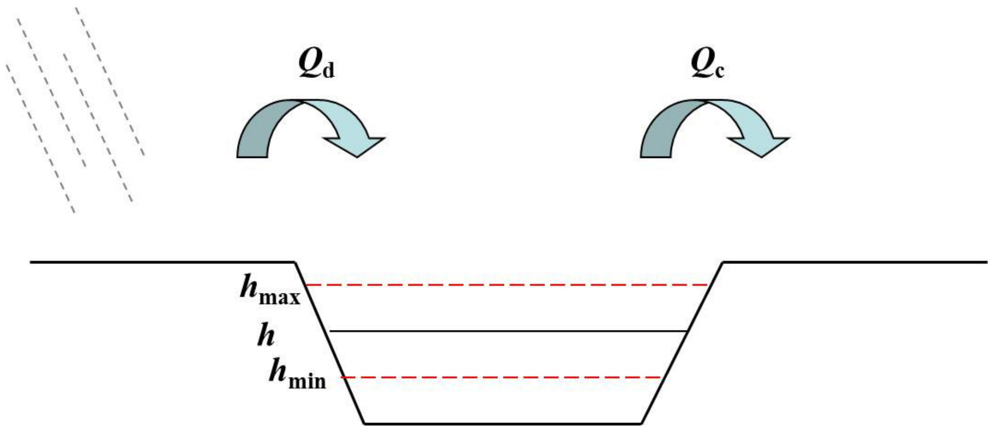

Fig. 1 Schematization of an ideal low-land water system

The dynamics of this water system can be described mathematically using the following model [2]:

$$h(k+1) = h(k) - \frac{Q_c(k)\Delta T}{A_s} + \frac{Q_d(k)\Delta T}{A_s} \tag{1}$$

where k represents the discrete time step index, ΔT (s) the time step, A_s (m^2) the area of the water system we consider, Q_c the flow out of the reservoir and Q_d the flow into the reservoir.

The change in flow, ΔQ_c (m^3/s), is the manipulated variable that we use to control the water level. It is defined as:

$$\Delta Q_{\mathrm{c}}(k) = Q_{\mathrm{c}}(k) - Q_{\mathrm{c}}(k-1) \tag{2}$$

3. THE CONTROL PROBLEM

As has been mentioned, we focus on three management objectives in this paper—navigation, safety and water demand. Specifically, we apply the model to the control of the low land water system, where the following objective function has to be optimized:

$$\begin{aligned} J(k) = \sum_{l=0}^{N-1} \{ & Q_{\mathrm{nv}} e^2(l+k) + Q_{\mathrm{sf}} e_{\max}{}^2(l+k) + Q_{\mathrm{wd}} e_{\min}{}^2(l+k) \\ & + R\Delta Q_{\mathrm{c}}{}^2(l+k) + R_{\max} u_{\max}{}^2(l+k) + R_{\min} u_{\min}{}^2(l+k) \} \\ & + 1000 * Q_{\mathrm{nv}} e^2(N) + Q_{\mathrm{sf}} e_{\max}{}^2(N) + Q_{\mathrm{wd}} e_{\min}{}^2(N) \\ & + R\Delta Q_{\mathrm{c}}{}^2(N) + R_{\max} u_{\max}{}^2(N) + R_{\min} u_{\min}{}^2(N) \end{aligned} \tag{3}$$

where $J(k)$ is the cost function to be optimized at step k; Q_{nv}, Q_{sf}, Q_{wd}, R, $R_{\max}$, $R_{\min}$ are technically called penalties on the control goals of navigation, safety, water demand and manipulated variables and e, $e_{\max}$, $e_{\min}$ are the deviations between the water level h and the reference, maximum allowed, minimum allowed levels h_{ref}, $h_{\max}$, $h_{\min}$ respectively:

$$\begin{aligned} & e(k)=h(k)-h_{\mathrm{ref}}, \\ & e_{\max}(k)=max\{h(k)-h_{\max},0\}= e(k)-u_{\max}(k), \\ & e_{\min}(k)=max\{h_{\min}- h(k),0\}= e(k)-u_{\min}(k). \end{aligned} \tag{4}$$

Here $e_{\max}$ and $u_{\max}$, e_{min} and $u_{\min}$ are actually introduced as soft constraints in order to avoid non-feasibility issues. The role of $e_{\max}$ and e_{min} is to show how much the water level exceeds the allowed range when extreme situations happen. In particular, $e_{\max}$ is either zero or a value corresponding with the exceeding of the maximum allowed level and e_{min} is either zero or a value outside the minimum allowed level with reversed sign. Besides that, $u_{\max}$ and $u_{\min}$ are virtual bounded inputs without physical meaning. Hence, $e_{\max}$ and $e_{\min}$ are zero whenever $u_{\max}$ and $u_{\min}$ are inside their boundaries. Otherwise, the values of these variables are different from zero, which is severely penalized in the cost function. As a consequence, the controller always tries to keep $e(k)$ within the gap corresponding to the bounds of $u_{\max}$ and $u_{\min}$. More details about the soft constraints can be found in [10].

Notice that the different terms of (3) are related to the different water management objectives that were presented in the introduction. A centralized MPC controller would minimize the cost function shown in (3) at each time step in order to calculate the corresponding control actions that will be taken over the next N time steps. Nevertheless, only the first one of these control actions is really implemented; the other actions are discarded. And the next time step, the procedure is repeated in a receding horizon fashion.

Centralized MPC merges all these goals in a single function, as was shown in (3). However, those problems are handled by different water boards in a distributed manner in reality. This paper illustrates how DMPC can be used to separate this multi-objective optimization into several components related with the corresponding objectives that have to be minimized by different parties. In this way, the computation can be carried out in parallel by the different parties that have a specific interest only in one of the goals. To this end, we will use dual decomposition [7], which allows us to overcome the coupling that may be present between the different objectives. As an example of such coupling, all the errors defined in (4) depend on the water level $h(k)$, which can be manipulated through $\Delta Q_{\mathrm{c}}(k)$. Hence, all the different optimization terms are coupled by this variable. In order to apply dual decomposition, we will allow each particular objective to choose its preferred value for this coupling variable. This leads to the following optimization problem:

$$\begin{aligned}
&\min_{\substack{\Delta Q_{\mathrm{c,nv}},\Delta Q_{\mathrm{c,sf}},\\ \Delta Q_{\mathrm{c,wd}},\Delta Q_{\mathrm{c,en}},\\ \Delta Q_{\mathrm{c}}}} \sum_{l=0}^{N} \{Q_{\mathrm{nv}}(l+k)e^2(l+k) + Q_{\mathrm{sf}}{e_{\max}}^2(l+k) + Q_{\mathrm{wd}}{e_{\min}}^2(l+k) \\
&\qquad + R\Delta {Q_{\mathrm{c}}}^2(l+k) + R_{\max}{u_{\max}}^2(l+k) + R_{\min}{u_{\min}}^2(l+k)\} \qquad (5)\\
&s.t.\\
&\Delta Q_{\mathrm{c,nv}}(l+k) = \Delta Q_{\mathrm{c,sf}}(l+k) = \Delta Q_{\mathrm{c,wd}}(l+k) = \Delta Q_{\mathrm{c}}(l+k), \forall l \in \{1,\ldots N-1\}
\end{aligned}$$

Note that the equality constraints impose that all the agents must agree on the value of the shared variable ΔQ_c(k). The Lagrangian multipliers of (5) transforms these coupling constraints into penalties on the difference between the different local versions of the manipulated variables, which allows us to distribute the optimization problem between the agents. These penalties are adjusted iteratively until all the agents share the same vision about the value of the coupling variable ΔQ_c(k). Thus, the use of the Lagrangian multipliers allows us to remove the equality constraints and to have a separable cost function [7]:

$$\begin{aligned}
&\max_{\substack{\lambda_{\mathrm{nv}},\\ \lambda_{\mathrm{sf}},\\ \lambda_{\mathrm{wd}}}} \min_{\substack{\Delta Q_{\mathrm{c,nv}},\\ \Delta Q_{\mathrm{c,sf}},\\ \Delta Q_{\mathrm{c,wd}},\\ \Delta Q_{\mathrm{c}}}} \sum_{l=0}^{N} \{Q_{\mathrm{nv}}(l+k)e^2(l+k) + Q_{\mathrm{sf}}{e_{\max}}^2(l+k) + Q_{\mathrm{wd}}{e_{\min}}^2(l+k) \\
&\qquad + R\Delta {Q_{\mathrm{c}}}^2(l+k) + R_{\max}{u_{\max}}^2(l+k) + R_{\min}{u_{\min}}^2(l+k) \\
&\qquad + \lambda_{\mathrm{nv}}(l+k)\left(\Delta Q_{\mathrm{c,nv}}(l+k) - \Delta Q_{\mathrm{c}}(l+k)\right) \qquad (6)\\
&\qquad + \lambda_{\mathrm{sf}}(l+k)\left(\Delta Q_{\mathrm{c,sf}}(l+k) - \Delta Q_{\mathrm{c}}(l+k)\right) \\
&\qquad + \lambda_{\mathrm{wd}}(l+k)\left(\Delta Q_{\mathrm{c,wd}}(l+k) - \Delta Q_{\mathrm{c}}(l+k)\right)\}
\end{aligned}$$

For example, the agent responsible for achieving the navigation goal would solve the following optimization problem for a given value of the Lagrangian multipliers:

$$\min_{\substack{\Delta Q_{\mathrm{c,nv}} \\ \lambda_{\mathrm{nv}}}} \sum_{l=0}^{N} (Q_{\mathrm{nv}} e(l+k)^2 + \lambda_{\mathrm{nv}}(k) \Delta Q_{\mathrm{c,nv}}) \tag{7}$$

After this problem is solved, the value of the Lagrangian multipliers is updated like in [7]. The procedure is repeated until convergence is attained, i.e., when the difference between the all the versions of ΔQ_c(k) is lower than a certain threshold (e.g. 10^{-3}).

4. SIMULATIONS

In this section we carry out a simulation study that illustrates the potential of the method proposed using the values shown in Table 1. We consider a simple scenario of six days, which starts with a storm event from hour 10 to hour 25 and high water demands from hour 60 to hour 80. The maximum disturbance due to rainfall can peak at 4000 m^3/s while the minimum disturbance due to high demand can reach a minimum of -1000 m^3/s. Remaining days keeps 100 m^3/s as regular daily input. A DMPC controller is used to regulate the outflow in order to maintain the water level as what we expect, where the maximum pump capacity is 800 m^3/s. The results are shown in Figure 2. Before the rain starts, the pump has already been turned on to create as much storage space as possible because of the coming storm in the next 12 hours. Note that the minimum water level is exceeded because of that. When the rain comes and peaks at 4000 m^3/s*,* the water only exceeds the maximum allowed level slightly, i.e., less than 0.2 m. After that, the water level declines gradually by pumping until we predict the high water demand in the coming hours. After hour 44, the pump is stopped so that the reservoir can store the expected daily rainfall to be well prepared for the water use in the next 20 hours. Actually, the water level only exceeds the minimum allowed water level less than 0.12 m and this violation lasts 9 hours only. As we have set the large penalty on the water level at the end of the horizon, it rises steadily until it reaches the set point.

Table1
System model and controller parameters

Parameters	Symbol	Value
Storage area	A_s	4e+7 (m^2)
Control time step	ΔT	3600 (s)
Reference/Maximum/Minimum Water level	$h_{ref}/h_{max}/ h_{min}$	0.4/1.5/-0.4 (m)
Prediction horizon	N	24 (1 day)
Penalties on $e/ e_{max}/ e_{min}$	$Q_{nv}/Q_{sf}/Q_{wd}$	10/100/10
Penalties on $\Delta Q_c/u_{max}/u_{min}$	$R/R_{max}/R_{min}$	1e-4/1e-6/1e-6

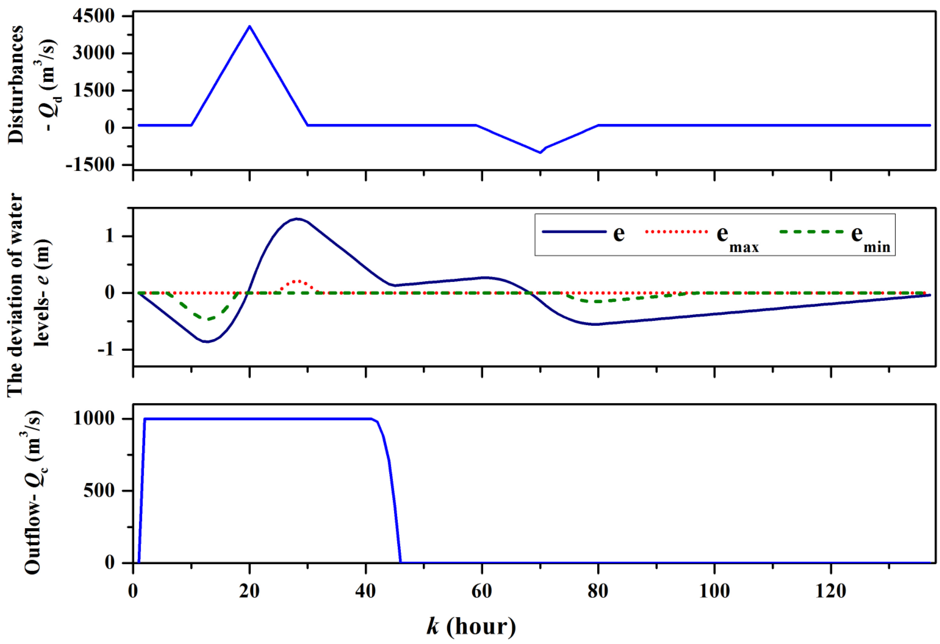

Fig. 2 Disturbances, water levels and the outflow

5. CONCLUSIONS

In this paper, we consider a water system management problem that has to meet three different objectives regarding navigation, safety and water demand. Distributed Model Predictive Control is presented as an approach for addressing this kind of multi-objective optimization problems. Future work focuses on applying this kind of approach for controlling large-scale water systems, such as the entire water system in the Netherlands.

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