Technical report

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P. Mc Namara, R.R. Negenborn, B. De Schutter, G. Lightbody. Coordination of a multiple link HVDC system using local communications based distributed model predictive control. In *Proceedings of the 18th IFAC World Congress* (IFAC WC'11), Milano, Italy, pp. 1558-1563, August-September 2011.

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Coordination of a multiple link HVDC system using local communications based Distributed Model Predictive Control

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Abstract: As the complexity of power networks increases, the installation of devices such as High Voltage Direct Current links (HVDC) and Flexible AC Transmission Systems (FACTS), and the use of advanced control techniques, can be used to improve network stability. Model Predictive Control (MPC) is an example of such an advanced control technique. However, it is often impractical to implement this technique in a centralised manner, as often the problem can be too computationally complex or several independent controllers may be responsible for different subsystems. Distributed approaches use communication between a number of controllers to approximate control of a centralised system. In this paper it is proposed to use distributed MPC for controlling a multiple link HVDC system using local communications only.

1. INTRODUCTION

Power networks are large, complex, highly interconnected systems. As increasing demands are imposed on power networks more advanced control techniques are needed in order to maintain network stability. HVDC lines allow for the efficient transmission of large quantities of power over long distances. Moreover, due to their high level of controllability these devices can improve transient stability and power system damping (Kundur, 1994). In Erikkson (2008), a multiple HVDC link system based on part of the Nordic power grid is presented. The techniques used to control this system were primarily centralised, non-optimization based control techniques.

Model Predictive Control (MPC) (Rossiter, 2003) (also known as Receding Horizon Control) is an optimisation based control technique, in which the controller uses state-space and output predictions to calculate optimal control moves for the system. One of the main advantages of this control technique, over non-optimization based techniques, is the systematic and intuitive manner in which constraints are incorporated into the control system and the fact that delays are naturally catered for. It is a mature technology at this stage, with stability and robustness analysis well established for the linear, time-invariant, centralised case.

Power system MPC problems can get quite large, and span vast geographical areas. Sections of these power systems can are often controlled by separate controllers, e.g. countries may share the a power network but will usually have their own control operator, or in a deregulated system many different companies may control the grid within a country. Thus, it is often desirable to use a control technique that allows a number of subsystems, using local controllers, to coordinate their actions.

Distributed MPC techniques allow these problems to be broken into a number of smaller local MPC problems that can be coordinated with communication. These methods have been seen to be quite effective when used for the control of power systems, many of which have included FACTS devices (Negenborn et al., 2008; Venkat, 2006; Talukdar et al., 2005). In this paper it is proposed to use distributed MPC using local communications for the control of the multiple HVDC link system. The distributed MPC technique used here (Negenborn et al., 2008) is extended to allow the coordination of the inputs, as well as states that are common to control problems of different agents.

2. THE MULTIPLE HVDC LINK SYSTEM

The continuous-time dynamics of the multiple link HVDC system under study here are described in this section.

2.1 Multiple HVDC link system description

Fig. 1 shows the system to which we will in this paper apply the generally applicable distributed MPC scheme discussed in the next section. This system is based on the multiple HVDC link system between Denmark, Norway, and Sweden (Erikkson, 2008). It consists of 4 buses with their own generation and loads. Both AC and HVDC lines connect the buses. The HVDC lines are of the Line Commutated Converter type (HVDC-LCC) (Pai et al., 1981). Generation capacities and loads are kept constant in this paper.

2.2 Modelling

The classical swing equations for generator a are:

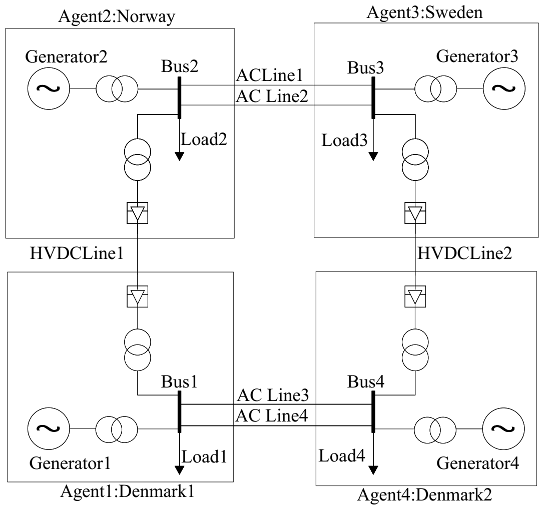


Fig. 1. The multiple HVDC link system with areas controlled by agents (Erikkson, 2008).

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta_{\mathrm{r},a}(t) = \omega_0 \Delta\omega_{\mathrm{r},a}(t) \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_{\mathrm{r},a}(t) = \frac{1}{2H_a}(P_{\mathrm{m},a}(t) - P_{\mathrm{G},a}(t) - D_a\Delta\omega_{\mathrm{r},a}(t)), \tag{2}$$

where $\delta_{\mathrm{r},a}(t)$ is the rotor angle (rad/s), H_a is the inertial constant (s), $\omega_{\mathrm{r},a}(t)$ is the rotor speed per unit, $\Delta\omega_{\mathrm{r},a}(t)$= $\omega_{\mathrm{r},a}(t)-1$ is the rotor speed deviation per unit, ω_0 is the base rotor speed (rad/s), $P_{\mathrm{m},a}(t)$ and $P_{\mathrm{G},a}(t)$ are the mechanical and generated power per unit, respectively, and D_a is the damping factor per unit (Kundur, 1994).

The current injected by generator a, $\overrightarrow{I}_{\mathrm{g},a}(t)$, is given by:

$$\overrightarrow{I}_{\mathrm{g},a}(t) = \frac{\overrightarrow{E}'_{\mathrm{q}a}(t) - \overrightarrow{U}_a(t)}{jx'_{\mathrm{d}a}}, \tag{3}$$

where $\overrightarrow{E}'_{\mathrm{q},a}(t) = E'_{\mathrm{q},a}(t)\angle\delta_a(t)$ is the internal voltage per unit with magnitude $E'_{\mathrm{q},a}(t)$ and angle $\delta_a(t)$, $\overrightarrow{U}_a(t) = U_a(t)\angle\theta_a(t)$ is the voltage per unit at the bus to which the generator is connected with magnitude $U_a(t)$ and angle $\theta_a(t)$, and $x'_{\mathrm{d},a}$ is the d-axis transient reactance. All variables are defined as in Kundur (1994). The generated power is then given by:

$$P_{\mathrm{G},a}(t) = \Re[\overrightarrow{E}_{\mathrm{g},a}(t)\,\overrightarrow{I}^*_{\mathrm{g},a}(t)]. \tag{4}$$

Equations (1) and (2) of the classical model of a synchronous generator assume that $E'_{\mathrm{q},a}(t)$ and $x'_{\mathrm{d},a}$ are constant (Kundur, 1994). These classical equations are suitable for analysis of power oscillations and transient stability studies. Note: For the sake of notational simplicity, below the continuous time index t is omitted.

An impedance matrix gives the relationship between the voltage nodes and currents in the system. Lines and loads are represented by impedances in this matrix.

A π-model representation (Kundur, 1994) of the AC lines is used to represent the line inductances and capacitances, including the possibility for 3-phase to ground faults in the middle of the lines. Loads are modelled as constant impedances in the impedance matrix, i.e. where $\overrightarrow{S}_a = P_{\mathrm{LL}a} + jQ_{\mathrm{LL}a}$ is the consumed complex load power in VA, the load impedance $\overrightarrow{X}_{\mathrm{LL}a}$ is given by

$$\overrightarrow{X}_{\mathrm{LL}a} = \frac{\overrightarrow{U}_a\overrightarrow{U}^*_a}{\overrightarrow{S}^*_a}, \tag{5}$$

where x^* denotes the complex conjugate of x.

The HVDC link model in Erikkson (2008) is used to simplify the representation of the system dynamics. This idealised version of the HVDC link assumes instantaneous, lossless power delivery and that the power factors are equal on both the inverter and rectifier sides. This model is further simplified by assuming that $Q_{\mathrm{HVDC},j} = q_{\mathrm{r},j}P_{\mathrm{HVDC},j}$, where $q_{\mathrm{r},j}$ is a constant, and $P_{\mathrm{HVDC},j}$ and $Q_{\mathrm{HVDC},j}$ are the active and reactive HVDC powers in HVDC line j (Pai et al., 1981).

The internal node representation is used to model the system dynamics (Erikkson, 2008) as it allows the power system to be represented using a system of first order differential equations. To do this it is assumed that $P_{\mathrm{m},a}$ is constant and that the loads are modelled as constant impedances.

Using Kirchoff's current law, an impedance matrix is constructed:

$$\begin{pmatrix} \boldsymbol{I}_{\mathrm{g}} \\ \boldsymbol{I}_{\mathrm{HVDC}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{Y}_{\mathrm{A}} & \boldsymbol{Y}_{\mathrm{B}} \\ \boldsymbol{Y}_{\mathrm{C}} & \boldsymbol{Y}_{\mathrm{D}} \end{pmatrix} \begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{U} \end{pmatrix}, \tag{6}$$

where $\boldsymbol{I}_{\mathrm{g}} = [\overrightarrow{I}_{\mathrm{g},1}, \ldots, \overrightarrow{I}_{\mathrm{g},n}]^{\mathrm{T}}$, $\boldsymbol{E} = [\overrightarrow{E}'_{\mathrm{q},1}, \ldots, \overrightarrow{E}'_{\mathrm{q},n}]^{\mathrm{T}}$, $\boldsymbol{U} = [\overrightarrow{U}_1, \ldots, \overrightarrow{U}_n]^{\mathrm{T}}$ and $\boldsymbol{I}_{\mathrm{HVDC}} = [\overrightarrow{I}_{\mathrm{HVDC},1,1}, \ldots, \overrightarrow{I}_{\mathrm{HVDC},n,m}]^{\mathrm{T}}$ where $\overrightarrow{I}_{\mathrm{HVDC},i,j}$ denotes the current coming from HVDC line i as seen by bus j.

From (6) the following can be found for $\boldsymbol{I}_{\mathrm{g}}$ in terms of $\boldsymbol{I}_{\mathrm{HVDC}}$ and $\boldsymbol{E}$:

$$\begin{aligned} \boldsymbol{I}_{\mathrm{g}} =& (\boldsymbol{Y}_{\mathrm{A}} - \boldsymbol{Y}_{\mathrm{B}}\boldsymbol{Y}_{\mathrm{D}}^{-1}\boldsymbol{Y}_{\mathrm{C}})\boldsymbol{E} + \boldsymbol{Y}_{\mathrm{B}}\boldsymbol{Y}_{\mathrm{D}}^{-1}\boldsymbol{I}_{\mathrm{HVDC}} \\ =& (\boldsymbol{G} + j\boldsymbol{B})\boldsymbol{E} + \boldsymbol{Y}_{\mathrm{HVDC}}\boldsymbol{I}_{\mathrm{HVDC}}. \end{aligned} \tag{7}$$

where $\boldsymbol{G}$=$\Re[\boldsymbol{Y}_{\mathrm{A}} - \boldsymbol{Y}_{\mathrm{B}}\boldsymbol{Y}_{\mathrm{D}}^{-1}\boldsymbol{Y}_{\mathrm{C}}]$, $\boldsymbol{B}$=$\Im[\boldsymbol{Y}_{\mathrm{A}} - \boldsymbol{Y}_{\mathrm{B}}\boldsymbol{Y}_{\mathrm{D}}^{-1}\boldsymbol{Y}_{\mathrm{C}}]$ and $\boldsymbol{Y}_{\mathrm{HVDC}}$=$\boldsymbol{Y}_{\mathrm{B}}\boldsymbol{Y}_{\mathrm{D}}^{-1}$.

Using (2), (4), and (7), yields the following swing equation for generator a:

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\omega_{\mathrm{r},a} = \frac{1}{2H_a}\Big(P_{\mathrm{m},a} - G_{a,a}E'^2_{\mathrm{q},a} - \\ &\sum_{\substack{l=1 \\ l\neq a}}^{n} E'_{\mathrm{q},a}E'_{\mathrm{q},l}(G_{a,l}\cos(\delta_{\mathrm{r},a} - \delta_{\mathrm{r},l}) + B_{a,l}\sin(\delta_{\mathrm{r},a} - \delta_{\mathrm{r},l})) \\ &\qquad + g_{a,1}P_{\mathrm{HVDC},1} \ldots + g_{a,m}P_{\mathrm{HVDC},m} - D_a\Delta\omega_{\mathrm{r},a}\Big) \end{aligned} \tag{8}$$

where $g_{a,j}$ is the coefficent of the contribution of the power injections from HVDC line j at bus a.

It is desired to maintain the rotor frequencies as close as possible to 1 pu at all times. Separate control operators are made responsible for the control of the 4 different areas. Therefore it is desirable to install a control system that returns the rotor frequencies to this setpoint in the minimum possible time after disturbances, in a distributed manner.

3. MODEL PREDICTIVE CONTROL

3.1 Definition of an agent

An agent is defined here as an entity responsible for the control of a system or subsystem, with access to the current state of the system or subsystem it controls. Agents have access to a model of the local system or subsystem and in the distributed case, agents are able to communicate with other agents who share a common variable. Agents compute values for their control inputs at discrete time steps based on the information available to them.

3.2 Description and state-space prediction

In MPC a control agent uses a discrete-time system model that predicts the system's future trajectory over a prediction horizon in order to calculate optimal discrete inputs over this horizon. Only the input for the first discrete time step is applied. At the next time step a new action is determined. The prediction horizon moves forward in a receding manner each time step.

A system consisting of n subsystems is considered, where each subsystem consists of a set of nodes and the interconnections between these nodes. Subsystems are assumed to be non-overlapping, i.e., nodes do not appear in 2 different subsystems. A discrete, linear, time-invariant state-space model is used to model the subsystem dynamics. This is given as follows:

$$\boldsymbol{x}_i(k+1) = A_i\boldsymbol{x}_i(k) + B_i\boldsymbol{u}_i(k) + D_i\boldsymbol{d}_i(k) + V_i\boldsymbol{v}_i(k) \quad (9)$$

$$\boldsymbol{y}_i(k) = C_i^{\mathrm{x}}\boldsymbol{x}_i(k) + C_i^{\mathrm{u}}\boldsymbol{u}_i(k) + C_i^{\mathrm{d}}\boldsymbol{d}_i(k) + C_i^{\mathrm{v}}\boldsymbol{v}_i(k), \quad (10)$$

where $\boldsymbol{x}_i(k)$ is the state of subsystem i, $\boldsymbol{u}_i(k)$ are local subsystem inputs, $\boldsymbol{d}_i(k)$ are known disturbances, $\boldsymbol{y}_i(k)$ are subsystem outputs, and $\boldsymbol{v}_i(k)$ are external inputs from other subsystems that influence subsystem i at sample time k.

To simplify notation, the prediction vector, of horizon N is first introduced. For a general vector $\boldsymbol{z}$, its prediction vector is $\tilde{\boldsymbol{z}}(k) = [\boldsymbol{z}^{\mathrm{T}}(k) \dots \boldsymbol{z}^{\mathrm{T}}(k+N-1)]^{\mathrm{T}}$. State predictions for subsystem i over the prediction horizon are then determined using (9) as follows:

$$\tilde{\boldsymbol{x}}_i(k+1) = \boldsymbol{A}_i^{\mathrm{f}}\boldsymbol{x}_i(k) + \boldsymbol{B}_i^{\mathrm{f}}\tilde{\boldsymbol{u}}_i(k) + \boldsymbol{D}_i^{\mathrm{f}}\tilde{\boldsymbol{d}}_i(k) + \boldsymbol{V}_i^{\mathrm{f}}\tilde{\boldsymbol{v}}_i(k) \quad (11)$$

where $\boldsymbol{A}_i^{\mathrm{f}}$, $\boldsymbol{B}_i^{\mathrm{f}}$, $\boldsymbol{D}_i^{\mathrm{f}}$, $\boldsymbol{V}_i^{\mathrm{f}}$ are the state space prediction matrices. The derivation of these matrices is well established in the literature (Rossiter, 2003).

3.3 MPC formulations

MPC for an individual subsystem: In a system of n subsystems, with agents i=1, …, n, assume a situation where agent i operates individually without communication with other agents. Suppose for now that it knows $\boldsymbol{x}_i(k)$, $\tilde{\boldsymbol{d}}_i(k)$, and $\tilde{\boldsymbol{v}}_i(k)$. The following optimisation problem is then solved at each time step:

$$\begin{aligned}\tilde{\boldsymbol{u}}_i(k) = &\arg\min_{\tilde{\boldsymbol{u}}_i(k)} J_i^{\mathrm{local}}(\boldsymbol{x}_i(k), \tilde{\boldsymbol{u}}_i(k), \tilde{\boldsymbol{d}}_i(k), \tilde{\boldsymbol{v}}_i(k)) \\ &\text{subject to } \tilde{\boldsymbol{u}}_i(k) \in \Omega_i,\ \tilde{\boldsymbol{x}}_i(k) \in \theta_i,\end{aligned} \quad (12)$$

where Ω_i and θ_i are the sets of admissable inputs and states, respectively, for subsystem i, and the local cost of subsystem i at the k^{th} sample time is (henceforth, denote $J_i^{\mathrm{local}}(\boldsymbol{x}_i(k), \tilde{\boldsymbol{u}}_i(k), \tilde{\boldsymbol{d}}_i(k), \tilde{\boldsymbol{v}}_i(k))$ as $J_i^{\mathrm{local}}(k)$, for convenience),

$$J_i^{\mathrm{local}}(k) = \sum_{p=0}^{N-1} J_i^{\mathrm{stage}}(k,p). \quad (13)$$

Here $J_i^{\mathrm{stage}}(k,p)$ is the cost at the p^{th} step of the prediction horizon for subsystem i at sample k. This is generally set up as a weighted sum of the square of the errors at the p^{th} prediction step.

Only the value for $\boldsymbol{u}_i(k)$ is applied to the subsystem after optimisation, and this process is repeated every time step, with the new prediction horizon moving forward one time step.

However, when many subsystems are interconnected, then knowledge of $\tilde{\boldsymbol{v}}_i(k)$ cannot be assumed, as $\tilde{\boldsymbol{v}}_i(k)$ is dependent on the dynamics of other subsystems. Hence, subsystems must reach a consensus on values for interconnecting variables. Before showing how to achieve consensus with MPC, terminology is developed to define interconnecting inputs and outputs.

Consider that there is a set of m_i agents, with indices $j \in \mathcal{N}_i$, which are connected to agent i. The interconnecting input vector, $\boldsymbol{w}_{ji}^{\mathrm{in}}$, is defined as the vector of inputs to control problem i from agent j and the interconnecting output vector $\boldsymbol{w}_{ji}^{\mathrm{out}}$ is defined as the vector of outputs to control problem j from agent i.

The vectors of all interconnecting inputs, $\tilde{\boldsymbol{w}}_i^{\mathrm{in}}$, and outputs, $\tilde{\boldsymbol{w}}_i^{\mathrm{out}}$, of agent i are given as follows:

$$\begin{aligned}\tilde{\boldsymbol{w}}_i^{\mathrm{in}} &= \tilde{\boldsymbol{v}}_i(k), \\ \tilde{\boldsymbol{w}}_i^{\mathrm{out}} &= \boldsymbol{E}_i[\tilde{\boldsymbol{x}}_i^{\mathrm{T}}(k+1) \quad \tilde{\boldsymbol{u}}_i^{\mathrm{T}}(k) \quad \tilde{\boldsymbol{y}}_i^{\mathrm{T}}(k)]^{\mathrm{T}}\end{aligned} \quad (14)$$

where $\boldsymbol{E}_i$ is a matrix of zeros except those places where a 1 picks out the appropriate variables shared with agents $j \in \mathcal{N}_i$.

Centralised MPC: In centralised MPC, instead of each subsystem having its own control agent, one central agent controls the whole system solving all the individual subsystems MPC problems simultaneously. For a system of n subsystems, the combined overall optimization problem is formed as follows:

$$\begin{aligned}&\min_{\tilde{\boldsymbol{u}}_1(k),\dots,\tilde{\boldsymbol{u}}_n(k)} \sum_{i=0}^{n} J_i^{\mathrm{local}}(k) \\ &\qquad \text{subject to } \tilde{\boldsymbol{u}}_i(k) \in \Omega_i,\ \tilde{\boldsymbol{x}}_i(k) \in \theta_i,\end{aligned} \quad (15)$$

and subject to the following equality constraints for $i = 1, \dots, n$,

$$\tilde{\boldsymbol{w}}_{ji}^{\mathrm{in}} = \tilde{\boldsymbol{w}}_{ij}^{\mathrm{out}}, \text{for } j \in \mathcal{N}_i, \quad (16)$$

i.e., all interconnecting variables are made equal to each other over the prediction horizon according to the dynamics of each subsystem as given in (11). This is usually a quadratic function that can be solved using a standard quadratic solver such as **quadprog** in Matlab.

However, often the implementation of centralised MPC can be impractical due to technical constraints, e.g., the computational load being too large. Therefore several agents are used to control different subsystems and the behaviour of these agents together approximates the behaviour of the centralised MPC.

Decentralised MPC: Decentralised MPC schemes assume that interconnected subsystems interact weakly and so ignore the effects of interactions with other subsystems in their MPC problems. Agents do not communicate with each other and independently solve an optimisation problem similar to (12) for each subsystem without seeking to achieve consensus amongst connected subsystems. However ignoring these interactions between subsystems can lead to highly suboptimal behaviour.

Distributed case: In distributed MPC systems agents communicate with each other in order to coordinate their control actions. An augmented Lagrangian formulation can be made of (15) to incorporate the equality constraints (16) into the cost function. In Negenborn et al. (2008) the quadratic terms of the augmented Lagrangian formulation are distributed across agents using Block Coordinate Descent (Royo, 2001).

In this method one agent at a time optimises values for its inputs, $\tilde{\boldsymbol{u}}_i(k)$, and its desired interconnecting input variables $\tilde{\boldsymbol{w}}_{ji}^{\text{in}}(k)$ for each $j \in \mathcal{N}_i$. The optimization problem of agent i, for $i = 1, \ldots, n$, for the l^{th} iteration of the distributed MPC cycle, at the k^{th} sample is:

$$\min_{\tilde{\boldsymbol{u}}_i(k), \{\tilde{\boldsymbol{w}}_{ji}^{\text{in}} : j \in \mathcal{N}_i\}} J_i^{\text{local}}(k) + \sum_{j \in \mathcal{N}_i} J_i^{\text{inter}}(k, l) \tag{17}$$

where $J_i^{\text{inter}}(k, l)$ is the cost associated with the inter-agent coordination given by:

$$\begin{aligned} J_i^{\text{inter}}(k, l) = &\begin{bmatrix} \tilde{\boldsymbol{\lambda}}_{ji}^{\text{in}}(l) \\ -\tilde{\boldsymbol{\lambda}}_{ij}^{\text{in}}(l) \end{bmatrix}^{\text{T}} \begin{bmatrix} \tilde{\boldsymbol{w}}_{ji}^{\text{in}}(l) \\ \tilde{\boldsymbol{w}}_{ji}^{\text{out}}(l) \end{bmatrix} \\ &+ \frac{c}{2} \left\| \begin{bmatrix} \tilde{\boldsymbol{w}}_{ij}^{\text{in,prev}}(l) - \tilde{\boldsymbol{w}}_{ji}^{\text{out}}(l) \\ \tilde{\boldsymbol{w}}_{ij}^{\text{out,prev}}(l) - \tilde{\boldsymbol{w}}_{ji}^{\text{in}}(l) \end{bmatrix} \right\|_2^2 . \end{aligned} \tag{18}$$

where c is a positive constant and $\tilde{\boldsymbol{\lambda}}_{ji}^{\text{in}}$ is the Lagrange multiplier associated with the interconnecting constraint $\tilde{\boldsymbol{w}}_{ji}^{\text{in}} = \tilde{\boldsymbol{w}}_{ij}^{\text{out}}$.

Each agent optimises this function, using a suitable optimisation package, in a serial fashion communicating the interconnecting variables with its neighbours. The values $\tilde{\boldsymbol{w}}_{ij}^{\text{out,prev}}(l)$, $\tilde{\boldsymbol{w}}_{ij}^{\text{in,prev}}(l)$ are taken as the most recently updated values of $\tilde{\boldsymbol{w}}_{ij}^{\text{out}}$ and $\tilde{\boldsymbol{w}}_{ij}^{\text{in}}$ respectively.

One optimisation cycle is completed when all agents have performed an optimisation. When the optimisation cycle is finished, Lagrange multipliers are updated as follows:

$$\tilde{\boldsymbol{\lambda}}_{ji}^{\text{in}}(l+1) = \tilde{\boldsymbol{\lambda}}_{ji}^{\text{in}}(l) + c\left(\tilde{\boldsymbol{w}}_{ji}^{\text{in}} - \tilde{\boldsymbol{w}}_{ij}^{\text{out}}\right), \tag{19}$$

Iterations are continued until:

$$\begin{gathered} ||\tilde{\boldsymbol{\lambda}}_{ji}^{\text{in}}(l+1) - \tilde{\boldsymbol{\lambda}}_{ji}^{\text{in}}(l)||_\infty \leq \epsilon \\ \text{for } i = 1, \ldots, n \text{ and } j \in \mathcal{N}_i \end{gathered} \tag{20}$$

where ϵ is a specified tolerance and $\|.\|_\infty$ denotes the infinity norm.

3.4 Extension to coupled inputs

In typical control applications, agents have their own local control inputs and control inputs are not shared between agents. However, in the application in this paper all 4 agents have to determine actions for the 2 control inputs, $P_{\text{HVDC},1}$ and $P_{\text{HVDC},2}$. In other circumstances different agents' local inputs may be coupled for example via the objective function or through the system dynamics.

In this paper agents achieve consensus on these shared control inputs by creating duplicate variable vector, $\tilde{\boldsymbol{w}}_{\text{u},a}$, for agent a, of the control inputs, $\tilde{\boldsymbol{u}}$. These duplicate variables are then treated as local control inputs by each of the agents. Equality constraints are then placed on the duplicate variables as follows $\tilde{\boldsymbol{w}}_{\text{u},1}=\tilde{\boldsymbol{w}}_{\text{u},2}$, $\tilde{\boldsymbol{w}}_{\text{u},2}=\tilde{\boldsymbol{w}}_{\text{u},3}, \ldots$, $\tilde{\boldsymbol{w}}_{\text{u},n-1}=\tilde{\boldsymbol{w}}_{\text{u},n}$, such that $\tilde{\boldsymbol{w}}_{\text{u},1}=\ldots=\tilde{\boldsymbol{w}}_{\text{u},n}$ for a system of n subsystems. When the problem is distributed amongst agents, then each agent will optimise to find the local duplicate inputs. Agents then compare their local duplicate inputs to the values calculated previously by connected agents' for their duplicate variables in order to achieve consensus.

When consensus is reached, i.e., agents have agreed on values for duplicate control inputs and other interconnecting variables, one agent is chosen per control input (the designation of these agents is problem dependent and would be chosen as seen fit by the parties involved in the control of the system) to apply its calculated value for the input to the real system. The input applied will differ slightly from that calculated by the other agents, depending on the values of c and ϵ, as these determine to what extent agents will form consensus on variables.

4. SIMULATION RESULTS

The distributed MPC scheme is used to control the coupled HVDC link system. The control inputs, the HVDC line powers, are common to all 4 agents and the AC connected buses share interconnecting variables too. A simplification here is to directly calculate and apply the HVDC powers. However in a real system, currents are injected and so these would have to be calculated from these powers.

One agent is assigned per HVDC link as the HVDC link control agent, sending the HVDC power it calculated at the end of each control cycle through the link. Communication is needed between the agents in order to coordinate the HVDC powers sent to and received by each agent.

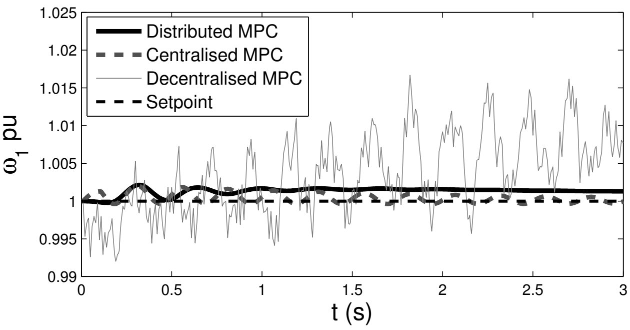
4.1 Simulation description and parameters

Simulations are carried out in Matlab 7.6. Simulink is used to simulate the nonlinear, continuous time power system. All MPC optimisations are performed using **quadprog**. Distributed and centralised MPC optimisations, using linearised state space models, are carried out at fixed time steps of 10ms using Matlab. A 3 phase to ground fault is applied to line 1 for 100ms after 1ms in the simulation. All output measurements are considered noise-free. The power system is set up as described in Section 2. The control system setup is described next.

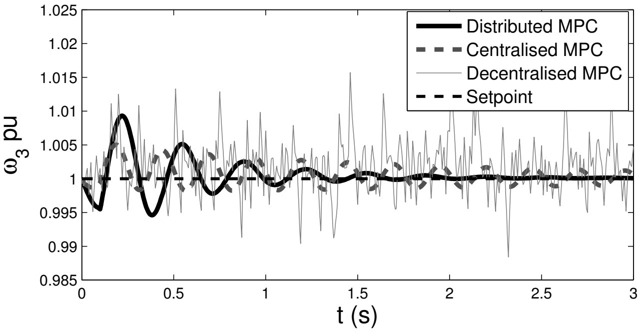
Linear, state-space control models of the subsystems are derived from (1) and (8), for each generator, in order to form state predictions. Before each control cycle the state equations are linearised about the current operating point as follows:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \Delta\delta_{\text{r},a} \\ \Delta\omega_{\text{r},a} \end{bmatrix} = &\begin{bmatrix} 0 & \omega_0 \\ \frac{\partial f_{\text{r},a}}{\partial \delta_{\text{r},a}}\big|_{op} & \frac{\partial f_{\text{r},a}}{\partial \omega_{\text{r},a}}\big|_{op} \end{bmatrix} \begin{bmatrix} \Delta\delta_{\text{r},a} \\ \Delta\omega_{\text{r},a} \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 \\ \frac{\partial f_{\text{r},a}}{\partial P_1}\big|_{op} & \frac{\partial f_{\text{r},a}}{\partial P_2}\big|_{op} \end{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\partial f_{\text{r},a}}{\partial \delta_{\text{r}l}}\big|_{op} \end{bmatrix} \Delta\delta_{\text{r}l} \end{aligned} \tag{21}$$

where Δ denotes a deviation of the relevant variable from its operating point, $f_{\text{r},a}(\delta_{\text{r},a}, \omega_{\text{r},a}, P_1, P_2, \delta_{\text{r}l}) = \frac{\mathrm{d}}{\mathrm{d}t}\omega_{\text{r},a}$, op denotes the current operating point for the relevant



(a) Plot of the frequency in area 1 vs time.



(b) Plot of the frequency in area 3 vs time.

Fig. 2. Sample plots of pu frequency vs. time.

variable and P_n is used instead of $P_{\mathrm{HVDC},n}$ for convenience in the above equation.

Taking the states $\boldsymbol{x}_a = [\Delta\delta_{\mathrm{r},a}\ \Delta\omega_{\mathrm{r},a}]^{\mathrm{T}}$, the inputs $\boldsymbol{u}_a = [\Delta P_{\mathrm{HVDC},1}\ \Delta P_{\mathrm{HVDC},2}]^{\mathrm{T}}$, the interconnecting input $\boldsymbol{v}_a = \Delta\delta_{\mathrm{r},l}$ and discretising equation (21) using Euler's method with a sample time $\tau = 0.01$s, the state-space equations can be formed as in (9). This is then used to make predictions for the distributed MPC controller. A prediction horizon of $N = 50$ is used so as to accurately represent the system dynamics in the optimisation.

Each agent a's stage cost function (there is one agent for each bus so for convenience the subscript a is used to index both), $J_a^{\mathrm{stage}}(k,p)$, for the p^{th} prediction step at sample step k, is given as follows:

$$J_a^{\mathrm{stage}}(k,p) = (\omega_{\mathrm{r},a}(k+p)-1)R_a(\omega_{\mathrm{r},a}(k+p)-1), \quad (22)$$

where $R_a = 0.8$. This cost function penalises deviations of the frequency from the base frequency.

The interconnection cost for the distributed MPC case at sample k and iteration l of the control cycle, $J_a^{\mathrm{inter}}(k,l)$, is formed from a centralised augmented Lagrangian MPC formulation which is given as follows:

$$\min_{\tilde{\boldsymbol{u}}_1(k),\ldots,\tilde{\boldsymbol{u}}_4(k)} \sum_{a=1}^{4}\left(J_a^{\mathrm{local}}(k)\right)$$
$$+\begin{bmatrix} \tilde{\boldsymbol{\lambda}}_{\mathrm{in}41}^{\delta_{\mathrm{r},4}} \\ \tilde{\boldsymbol{\lambda}}_{\mathrm{in}32}^{\delta_{\mathrm{r},3}} \\ \tilde{\boldsymbol{\lambda}}_{\mathrm{in}23}^{\delta_{\mathrm{r},2}} \\ \tilde{\boldsymbol{\lambda}}_{\mathrm{in}14}^{\delta_{\mathrm{r},1}} \\ \tilde{\boldsymbol{\lambda}}_{\mathrm{u},41} \\ \tilde{\boldsymbol{\lambda}}_{\mathrm{u},12} \\ \tilde{\boldsymbol{\lambda}}_{\mathrm{u},23} \\ \tilde{\boldsymbol{\lambda}}_{\mathrm{u},34} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \tilde{\boldsymbol{w}}_{\mathrm{in}41}^{\delta_{\mathrm{r},4}} - \tilde{\boldsymbol{w}}_{\mathrm{out}14}^{\delta_{\mathrm{r},4}} \\ \tilde{\boldsymbol{w}}_{\mathrm{in}32}^{\delta_{\mathrm{r},3}} - \tilde{\boldsymbol{w}}_{\mathrm{out}23}^{\delta_{\mathrm{r},3}} \\ \tilde{\boldsymbol{w}}_{\mathrm{in}23}^{\delta_{\mathrm{r},2}} - \tilde{\boldsymbol{w}}_{\mathrm{out}32}^{\delta_{\mathrm{r},2}} \\ \tilde{\boldsymbol{w}}_{\mathrm{in}14}^{\delta_{\mathrm{r},1}} - \tilde{\boldsymbol{w}}_{\mathrm{out}41}^{\delta_{\mathrm{r},1}} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},1} - \tilde{\boldsymbol{w}}_{\mathrm{u},4} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},2} - \tilde{\boldsymbol{w}}_{\mathrm{u},1} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},3} - \tilde{\boldsymbol{w}}_{\mathrm{u},2} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},4} - \tilde{\boldsymbol{w}}_{\mathrm{u},3} \end{bmatrix} + \frac{c}{2}\left\| \begin{matrix} \tilde{\boldsymbol{w}}_{\mathrm{in}41}^{\delta_{\mathrm{r},4}} - \tilde{\boldsymbol{w}}_{\mathrm{out}14}^{\delta_{\mathrm{r},4}} \\ \tilde{\boldsymbol{w}}_{\mathrm{in}32}^{\delta_{\mathrm{r},3}} - \tilde{\boldsymbol{w}}_{\mathrm{out}23}^{\delta_{\mathrm{r},3}} \\ \tilde{\boldsymbol{w}}_{\mathrm{in}23}^{\delta_{\mathrm{r},2}} - \tilde{\boldsymbol{w}}_{\mathrm{out}32}^{\delta_{\mathrm{r},2}} \\ \tilde{\boldsymbol{w}}_{\mathrm{in}14}^{\delta_{\mathrm{r},1}} - \tilde{\boldsymbol{w}}_{\mathrm{out}41}^{\delta_{\mathrm{r},1}} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},1} - \tilde{\boldsymbol{w}}_{\mathrm{u},4} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},2} - \tilde{\boldsymbol{w}}_{\mathrm{u},1} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},3} - \tilde{\boldsymbol{w}}_{\mathrm{u},2} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},4} - \tilde{\boldsymbol{w}}_{\mathrm{u},3} \end{matrix} \right\|_2^2 \quad (23)$$

where l's are omitted for compactness. This formulation enables the distribution of the problem in such a way that agents can reach agreement on the control inputs, i.e., the HVDC powers.

Each agent a has a duplicate vector of the control inputs $\tilde{\boldsymbol{w}}_{\mathrm{u},a}(k) = [\Delta\tilde{\boldsymbol{P}}_{\mathrm{HVDC},1}(k)^{\mathrm{T}}\ \Delta\tilde{\boldsymbol{P}}_{\mathrm{HVDC},2}(k)]$. The order in which agents optimise for the distributed MPC cycles starts with agent 1 and ends with 4. Therefore in the centralised case the equality constraint $\tilde{\boldsymbol{w}}_{\mathrm{u},a}(k) = \tilde{\boldsymbol{w}}_{\mathrm{u},a,\mathrm{last}}(k)$ is applied for each agent ($w_{\mathrm{u},a,\mathrm{last}}$ denotes the last agent to optimise) in order to reach consensus on the duplicate input values. Interconnecting constraints between interconnecting rotor position variables, δ_{r}, are also applied.

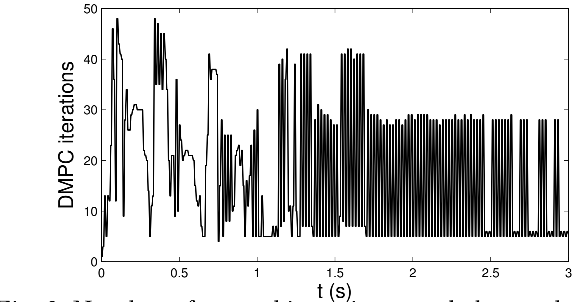


Fig. 3. Number of control iterations needed at each sample.

When (23) is distributed amongst the agents using Block Coordinate Descent, $J_a^{\mathrm{inter}}(k,l)$, takes the following distributed form for agent a, where bus j is AC connected to bus a:

$$J_a^{\mathrm{inter}} = \begin{bmatrix} \tilde{\boldsymbol{\lambda}}_{\mathrm{in},j,a}^{\delta_{\mathrm{r},j}} \\ -\tilde{\boldsymbol{\lambda}}_{\mathrm{in},a,j}^{\delta_{\mathrm{r},a}} \\ \tilde{\boldsymbol{\lambda}}_{\mathrm{u},a} \\ -\tilde{\boldsymbol{\lambda}}_{\mathrm{u},a,\mathrm{next}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \tilde{\boldsymbol{w}}_{\mathrm{in},j,a}^{\delta_{\mathrm{r},j}} \\ \tilde{\boldsymbol{w}}_{\mathrm{out},j,a}^{\delta_{\mathrm{r},a}} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},a} \\ \tilde{\boldsymbol{w}}_{\mathrm{u},a} \end{bmatrix} + \frac{c}{2}\left\| \begin{matrix} \tilde{\boldsymbol{w}}_{\mathrm{out},a,j}^{\delta_{\mathrm{r},j},\mathrm{prev}} - \tilde{\boldsymbol{w}}_{\mathrm{in},j,a}^{\delta_{\mathrm{r},j}} \\ \tilde{\boldsymbol{w}}_{\mathrm{in},a,j}^{\delta_{\mathrm{r},a},\mathrm{prev}} - \tilde{\boldsymbol{w}}_{\mathrm{out},j,a}^{\delta_{\mathrm{r},a}} \\ \tilde{\boldsymbol{w}}_{\mathrm{u,last}}^{\mathrm{prev}} - \tilde{\boldsymbol{w}}_{\mathrm{u},a} \\ \tilde{\boldsymbol{w}}_{\mathrm{u,next}}^{\mathrm{prev}} - \tilde{\boldsymbol{w}}_{\mathrm{u},a} \end{matrix} \right\|_2^2 \quad (24)$$

where $w_{\mathrm{u},a,\mathrm{next}}$ denotes the next agent to optimise and k's and l's are dropped for compactness.

After agent a optimises, it sends the relevant updated values of the variables to the agents to which it is connected for use in their distributed MPC optimisations. The total cost function for agent a is given by (17). This can be put into quadratic form using simple matrix manipulation where the optimisation vector is $\tilde{\boldsymbol{u}}_{opt}(k) = [\tilde{\boldsymbol{u}}^{\mathrm{T}}(k)\ \tilde{\boldsymbol{w}}_{\mathrm{in}}^{\mathrm{T}}(k)]^{\mathrm{T}}$. This is a vector of 149 variables representing the HVDC powers and interconnecting inputs to each area over the full prediction horizon.

The HVDC lines range in per unit are $-\mathbf{2} \leq \tilde{\boldsymbol{P}}_{\mathrm{HVDC}}(k) \leq \mathbf{2}$ and the per unit frequency range at all buses is $\mathbf{0.99} \leq \tilde{\boldsymbol{\omega}}(k) \leq \mathbf{1.01}$, where $\mathbf{A}=[A,\ldots,A]^{\mathrm{T}}$. The distributed MPC parameters related to communication are given as follows: $c = 0.1$, $\epsilon = 10^{-4}$.

When the distributed MPC control iterations are completed the final control inputs, $P_{\mathrm{HVDC},1}(k)$ and $P_{\mathrm{HVDC},2}(k)$ calculated by agents 2 and 3 respectively, are the control inputs that are applied.

scheme	MPC		
	Decentralised	Centralised	Distributed
J_{sim}	0.0065	0.0023	0.0045

Table 1. Comparison of J_{sim} for MPC schemes.

4.2 Results

The results of the simulation run can be seen in the Figs. 2(a) and 2(b), which show the frequencies at buses 1 and 3 plotted against time. These results are compared with centralised and decentralised MPC controllers. The cost over the full simulation is computed as follows:

$$J_{\mathrm{sim}} = \sum_{a=1}^{n} \sum_{k=1}^{t_{\mathrm{f}}} J_a^{\mathrm{stage}}(k). \tag{25}$$

where t_{f} is the final sample in the simulation. The state values used in calculating $J_a^{\mathrm{stage}}(k)$ are the state values taken from the power system at the k^{th} sample measured during simulation. It can be seen in Table 1 that the J_{sim} performance of the distributed MPC lies between that of the decentralised MPC and the centralised MPC.

Looking at Figs. 2(a) and 2(b), the frequency responses obtained with distributed MPC are quite close to those of the centralised controller and much better than those found with the decentralised controller. The trade-off for this performance is a significant computational and communications overhead.

The average and longest times for a full distributed MPC cycle (i.e. the time taken for agents to reach their final decisions with consensus on the interconnecting variables) were 1.23 s and 3.17 s respectively on a computer with an Intel® Core™ 2 6400 operating at 2.13GHz and with 3 GB of RAM. Each agent communicates to it's connected agents once in a serial fashion during during a distributed MPC iteration.

The number of distributed MPC iterations necessary to complete each optimisation cycle (i.e. the number of iterations needed at each distributed MPC cycle for all agents to reach consensus) represents the level of communication necessary at each sample. This is given in Fig 3.

The disparity between centralised and distributed MPC setpoint tracking performance could be accounted for by the fact the distributed MPC optimises for a set of duplications of the control inputs and only seeks agreement on these duplications within ϵ in order to calculate the suitable control inputs. The centralised MPC calculates only one set of control inputs which are applied to the system and so the predictions for each area are more accurate, resulting in better setpoint tracking performance.

In the example in this paper, generation capacities are kept constant and the modulation of the HVDC links alone is used to restabilise the system. System performance could potentially be improved by allowing generator capacities to vary, as is the case in most real systems. The improved controllability could allow offsets such as those in Fig. 2(a) to be reduced.

5. CONCLUSIONS AND FUTURE RESEARCH

Here the application of a distributed MPC to a multiple link HVDC system is proposed. The resultant setpoint tracking performance is significantly better than that of a decentralised MPC controller and close to that of a centralised MPC controller.

However this performance comes with a significant communication and computational overhead. Ways of reducing the computational and communication overhead will be needed before the distributed MPC in this paper could be implemented in reality. Furthermore, stability and convergence guarantees should be investigated for this distributed MPC technique. Communication delays and data transmission errors are other issues that would affect the control performance. These problems form the basis for future research in this area.

ACKNOWLEDGMENTS

The author would like to thank Dr. Mick Egan for his help regarding power systems theory. This work was funded by the Irish Research Council for Science, Engineering and Technology (IRCSET) and supported by the BSIK project "Next Generation Infrastructures (NGI)", the Delft Research Center Next Generation Infrastructures, the European STREP project "Hierarchical and distributed model predictive control (HD-MPC)", contract number INFSO-ICT-223854, and the VENI project "Intelligent multi-agent control for flexible coordination of transport hubs" (project 11210) of the Dutch Technology Foundation STW.

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