Robot Localization and Kalman Filters

Rudy Negenborn
rudy@negenborn.net

August 26, 2003

Outline

- Robot Localization
- Probabilistic Localization
- Kalman Filters
- Kalman Localization
- Kalman Localization with Landmarks

Robot Localization

- Localization a key problem
- Available location information
  - Relative Measurements
    - Driving: wheel encoders, accerelometers, gyroscopes
    - Frequent, but increasing error
  - Absolute Measurements
    - Sensing: GPS, vision, laser, landmarks
    - Less frequent, but bounded error

Probabilistic Localization

- Probabilistic approach
  - Consider whole space of locations
- Belief
  - $Bel(x_k) = P(x_k \mid d_1, \ldots, d_k)$
  - Get belief as close to real distribution as possible
  - Prior Belief
    - $Bel^-(x_k) = P(x_k \mid z_1, a_1, \ldots, z_{k-1}, a_{k-1})$
  - Posterior Belief
    - $Bel^+(x_k) = P(x_k \mid z_1, a_1, \ldots, z_{k-1}, a_{k-1}, z_k)$

Probabilistic Localization

- Localization equations:
  - $\text{Bel}^-(x_k) = P(x_k \mid z_1, a_1, ..., z_{k-1}, a_{k-1})$
    $= \int P(x_k \mid a_{k-1}, x_{k-1}) \cdot \text{Bel}^+(x_{k-1})\, dx_{k-1}$
  - $\text{Bel}^+(x_k) = P(x_k \mid z_1, a_1, \ldots, z_{k-1}, a_{k-1}, z_k)$
    $$= \frac{P(z_k \mid x_k)\, \text{Bel}^-(x_k)}{P(z_k \mid z_1, a_1, \ldots, z_{k-1}, a_{k-1})}$$

Markov Assumption

- Implementation Issues:
  - Motion model: $P(x_k \mid a_{k-1}, x_{k-1})$
  - Measurement model: $P(z_k \mid x_k)$
  - Representation of belief

Kalman Filters

- Representation of belief
  - Gaussian function
  - Mean and (co)variance
  - Initial belief: $Bel(x_0) = N(x_0, P_0)$
- Motion model
  - $x_k = Ax_{k-1} + Ba_{k-1} + w_{k-1}$, where $w_k \sim N(0, Q_k)$
- Measurement model
  - $z_k = Hx_k + v_k$, where $v_k \sim N(0, R_k)$

Kalman Filters

- Representation of belief
  - Gaussian function
  - Mean and (co)variance
  - Initial belief: $Bel(x_0) = N(x_0, P_0)$
- Motion model
  - $x_k = Ax_{k-1} + Ba_{k-1} + w_{k-1}$, where $w_k \sim N(0, Q_k)$
  - $P(x_k \mid a_{k-1}, x_{k-1}) = N(Ax_{k-1}, Q_k)$
- Measurement model
  - $z_k = Hx_k + v_k$, where $v_k \sim N(0, R_k)$
  - $P(z_k \mid x_k) = N(Hx_k, R_k)$

Kalman Filters

- Prior belief: $Bel^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$

- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$

Kalman Filters

- Prior belief: $Bel^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$
  - Prior location estimate: $\hat{x}_k^{-}$
  - Prior uncertainty: P_k^{-}
- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$
  - Posterior location estimate: $\hat{x}_k^{+}$
  - Posterior uncertainty: P_k^{+}

Kalman Filters

- Prior belief: $Bel^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$
  - $\hat{x}_k^{-} = A \cdot \hat{x}_{k-1}^{+} + B \cdot \hat{a}_{k-1}$
  - $P_k^{-} = A \cdot P_{k-1}^{+} \cdot A^T + B \cdot U_{k-1} \cdot B^T + Q_{k-1}$

Kalman Filters

- Prior belief: $Bel^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$
  - $\hat{x}_k^{-} = A \cdot \hat{x}_{k-1}^{+} + B \cdot \hat{a}_{k-1}$

    Prior location estimate

    Posterior location estimate

    Last relative measurement
  - $P_k^{-} = A \cdot P_{k-1}^{+} \cdot A^T + B \cdot U_{k-1} \cdot B^T + Q_{k-1}$

Kalman Filters

- Prior belief: $Bel^{-}(x_k) = N(\hat{x}_k^{-}, P_k^{-})$
  - $\hat{x}_k^{-} = A \cdot \hat{x}_{k-1}^{+} + B \cdot \hat{a}_{k-1}$
    - Prior location estimate
    - Posterior location estimate
    - Last relative measurement
  - $P_k^{-} = A \cdot P_{k-1}^{+} \cdot A^T + B \cdot U_{k-1} \cdot B^T + Q_{k-1}$
    - Prior uncertainty
    - Posterior uncertainty
    - Relative measurement uncertainty
    - Motion uncertainty

Kalman Filters

- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$
  - $\hat{x}_k^{+} = \hat{x}_k^{-} + K_k \cdot (z_k - H \cdot \hat{x}_k^{-})$
  - $K_k = P_k^{-} \cdot H^T \cdot (H \cdot P_k^{-} \cdot H^T + R_k)^{-1}$
  - $P_k^{+} = (I - K_k \cdot H) \cdot P_k^{-}$

Kalman Filters

- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$

  - $$\hat{x}_k^{+} = \hat{x}_k^{-} + K_k \cdot (z_k - H \cdot \hat{x}_k^{-})$$

    Residual: $(z_k - H \cdot \hat{x}_k^{-})$

    Posterior state estimate: $\hat{x}_k^{+}$; Prior state estimate: $\hat{x}_k^{-}$; Kalman Gain: K_k; True measurement: z_k; Measurement prediction: $H \cdot \hat{x}_k^{-}$

  - $$K_k = P_k^{-} \cdot H^T \cdot (H \cdot P_k^{-} \cdot H^T + R_k)^{-1}$$

  - $$P_k^{+} = (I - K_k \cdot H) \cdot P_k^{-}$$

Kalman Filters

- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$

  - $\hat{x}_k^{+} = \hat{x}_k^{-} + K_k \cdot (z_k - H \cdot \hat{x}_k^{-})$

    Residual

    Posterior state estimate

    Prior state estimate

    Kalman Gain

    True measurement

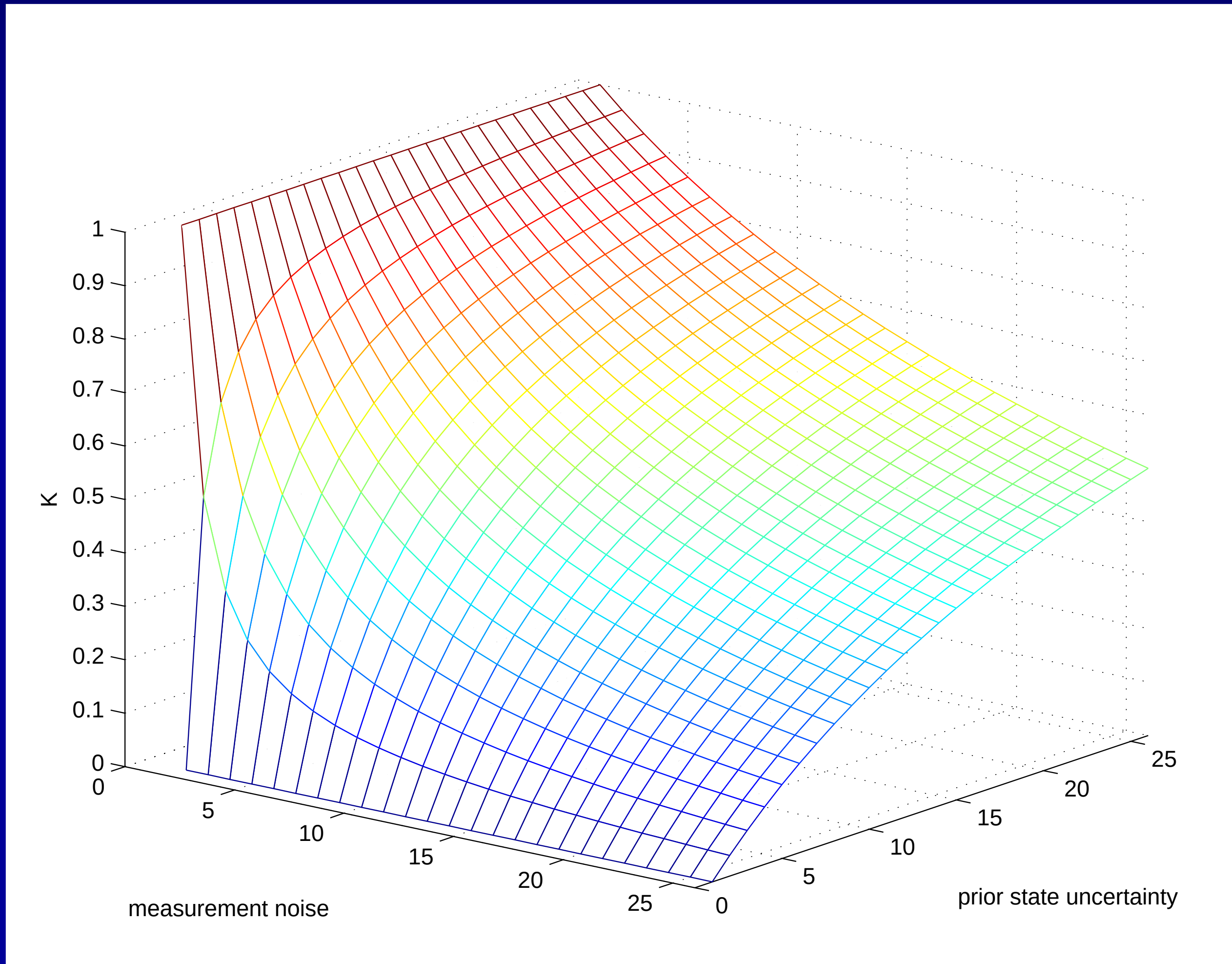
    Measurement prediction

  - $K_k = P_k^{-} \cdot H^{T} \cdot (H \cdot P_k^{-} \cdot H^{T} + R_k)^{-1}$

    Measurement residual uncertainty

  - $P_k^{+} = (I - K_k \cdot H) \cdot P_k^{-}$

Kalman Gain


K
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
measurement noise
0
5
10
15
20
25
prior state uncertainty

Kalman Filters

- Posterior belief: $Bel^{+}(x_k) = N(\hat{x}_k^{+}, P_k^{+})$
  - $\hat{x}_k^{+} = \hat{x}_k^{-} + K_k \cdot (z_k - H \cdot \hat{x}_k^{-})$

Residual

Posterior state estimate

Prior state estimat

Kalman Gain

True measurement

Measurement prediction

- $K_k = P_k^{-} \cdot H^T \cdot (H \cdot P_k^{-} \cdot H^T + R_k)^{-1}$

Measurement residual uncertainty

- $P_k^{+} = (I - K_k \cdot H) \cdot P_k^{-}$

Extended Kalman Filter

- Nonlinear motion and measurement models

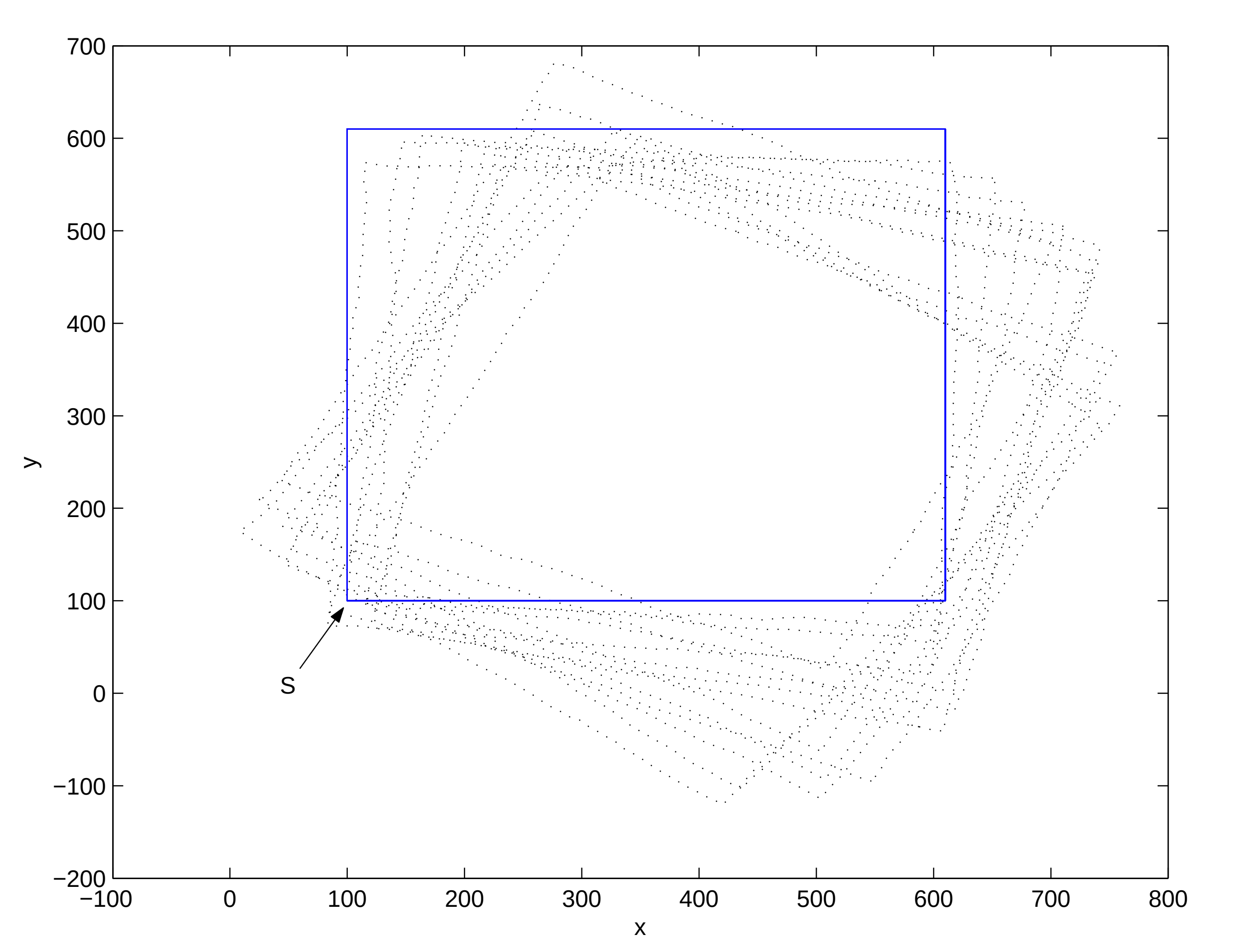
- Linearization around estimated trajectory
  - Partial derivatives of nonlinear model for A, B, H
  - Close to linear over uncertainty region

- Drawbacks
  - Evaluation at every time step
  - Linearization errors

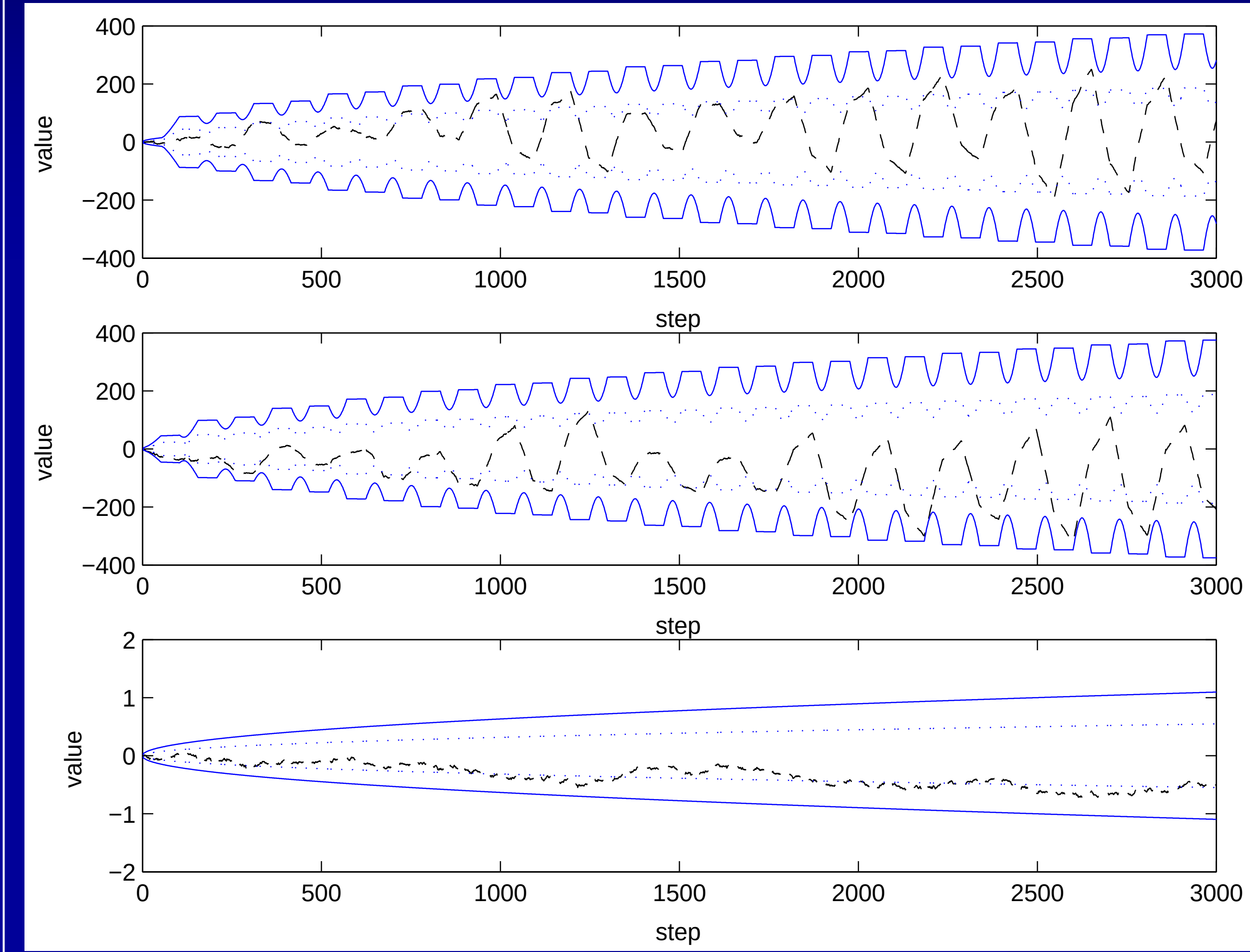
Kalman Localization

- Localization instances
  - Position Tracking
    - Initial belief with peak at true initial location
  - Global Localization
    - Initial uniform belief
  - Kidnapped Robot
    - Initial belief with peak far from true location

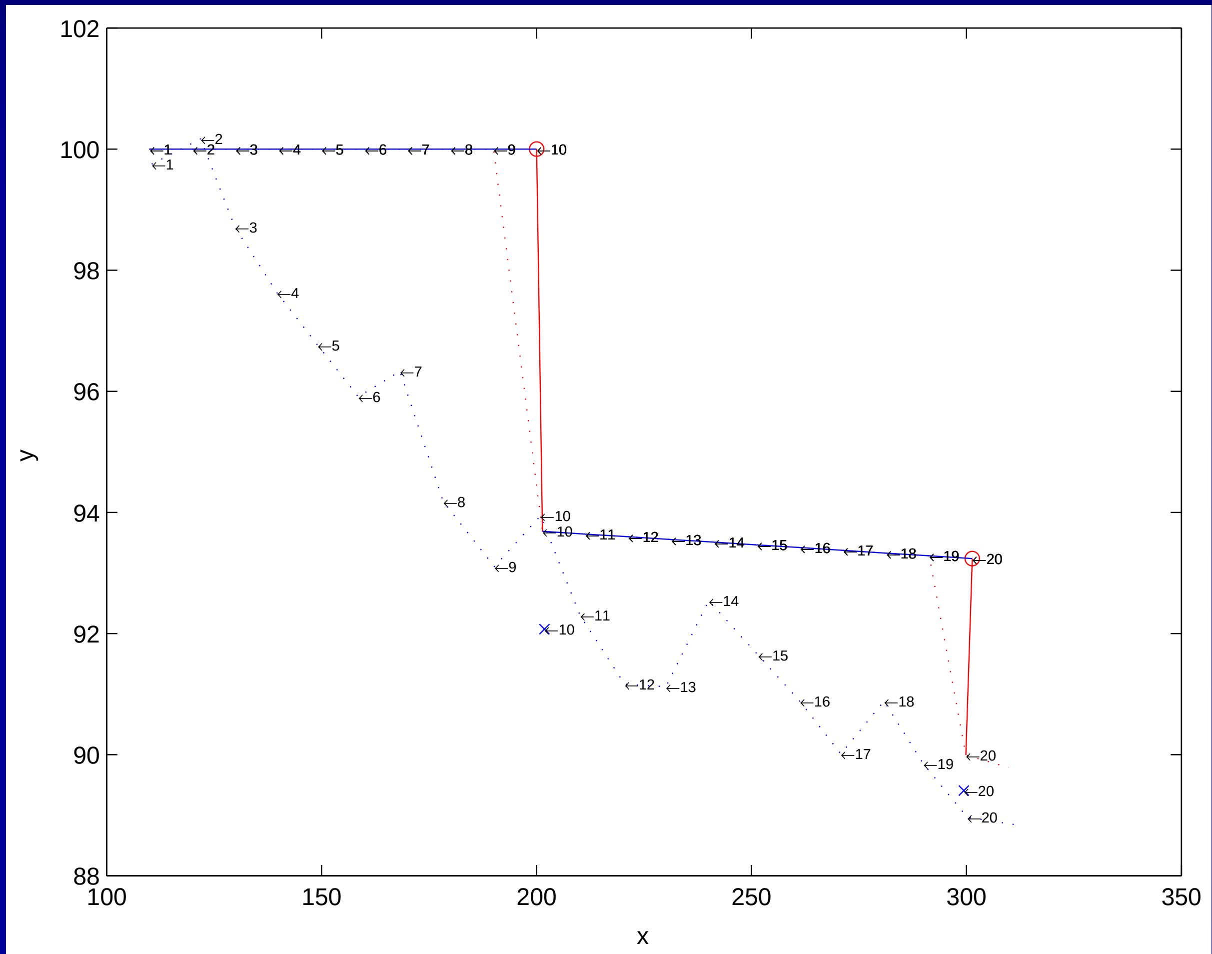
Position Tracking



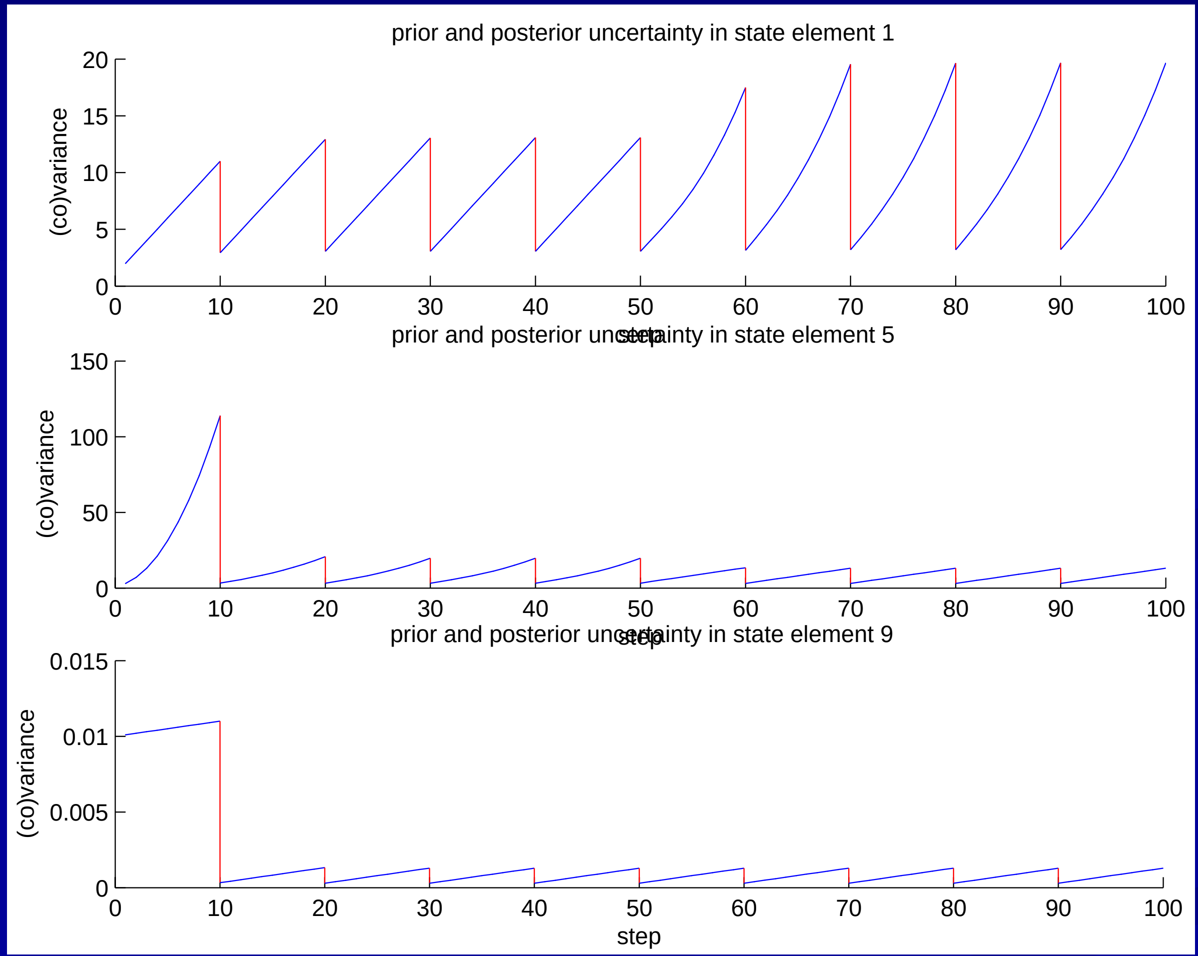
Position Tracking



Position Tracking



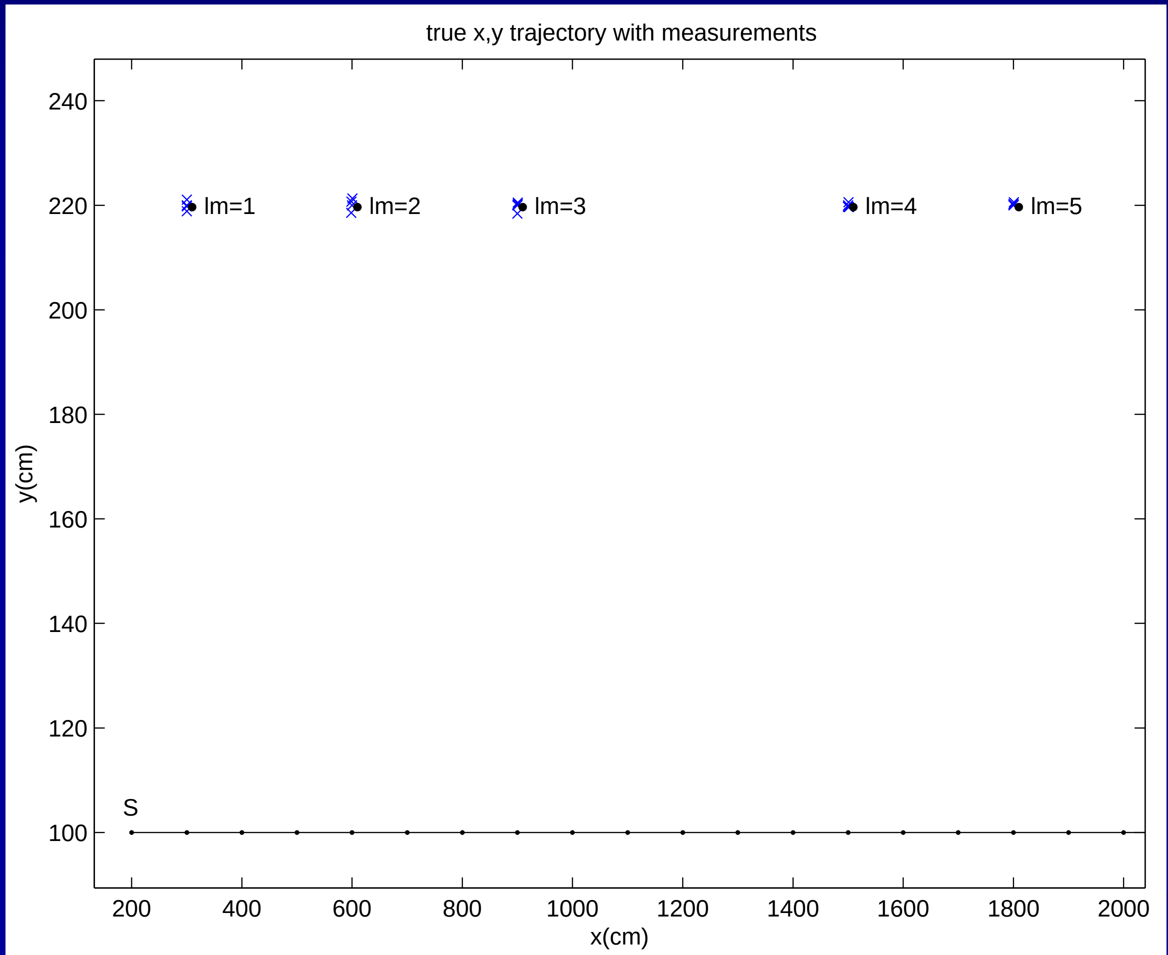
Infrequent Measurements



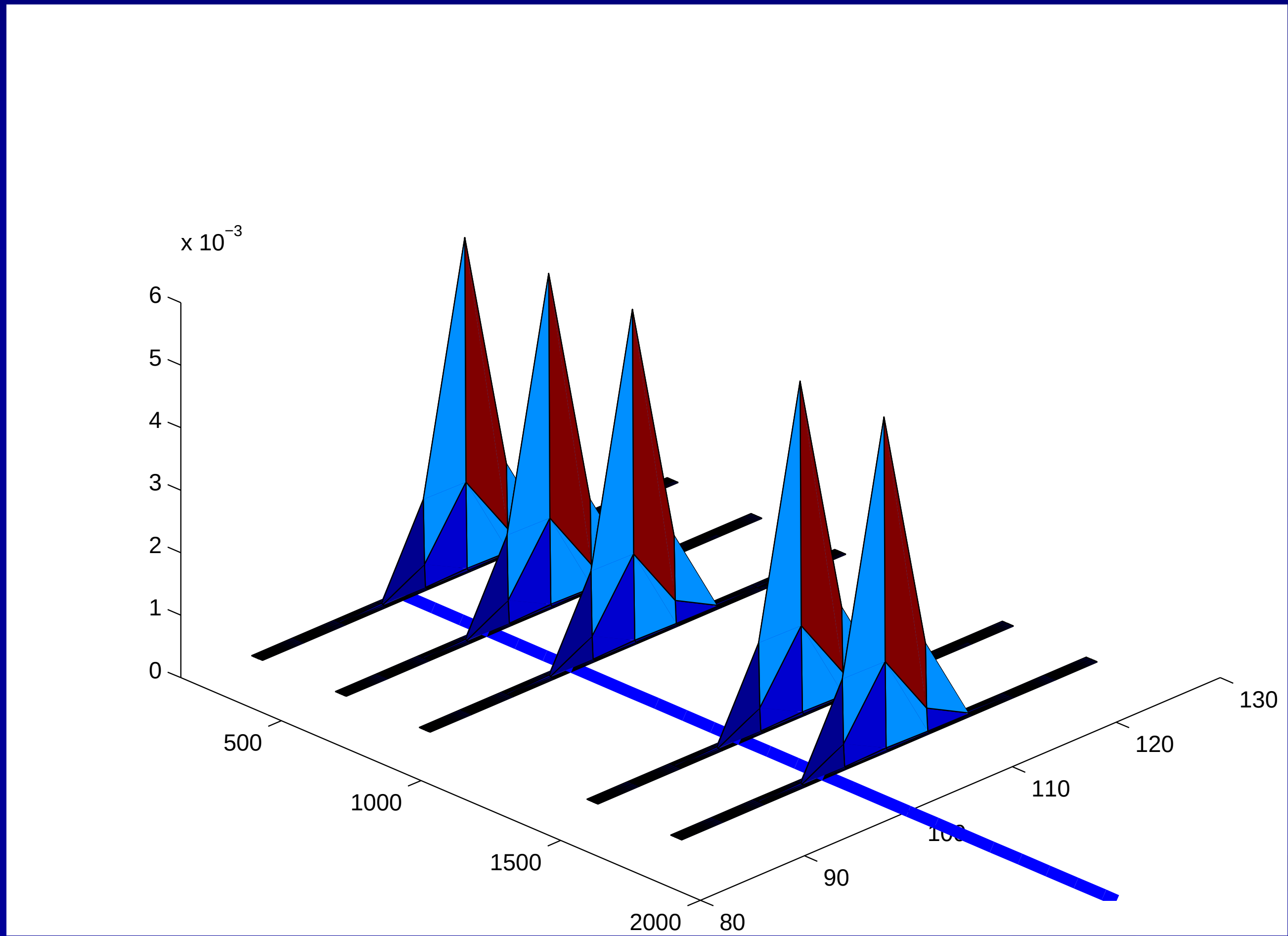
Kalman Localization with Landmarks

- Uniquely identifiable landmarks
  - 1:1 correspondence
- Type identifiable landmarks
  - 1:n correspondence
  - Kalman Filter framework extension
    - Multiple state beliefs
    - Probability for each belief

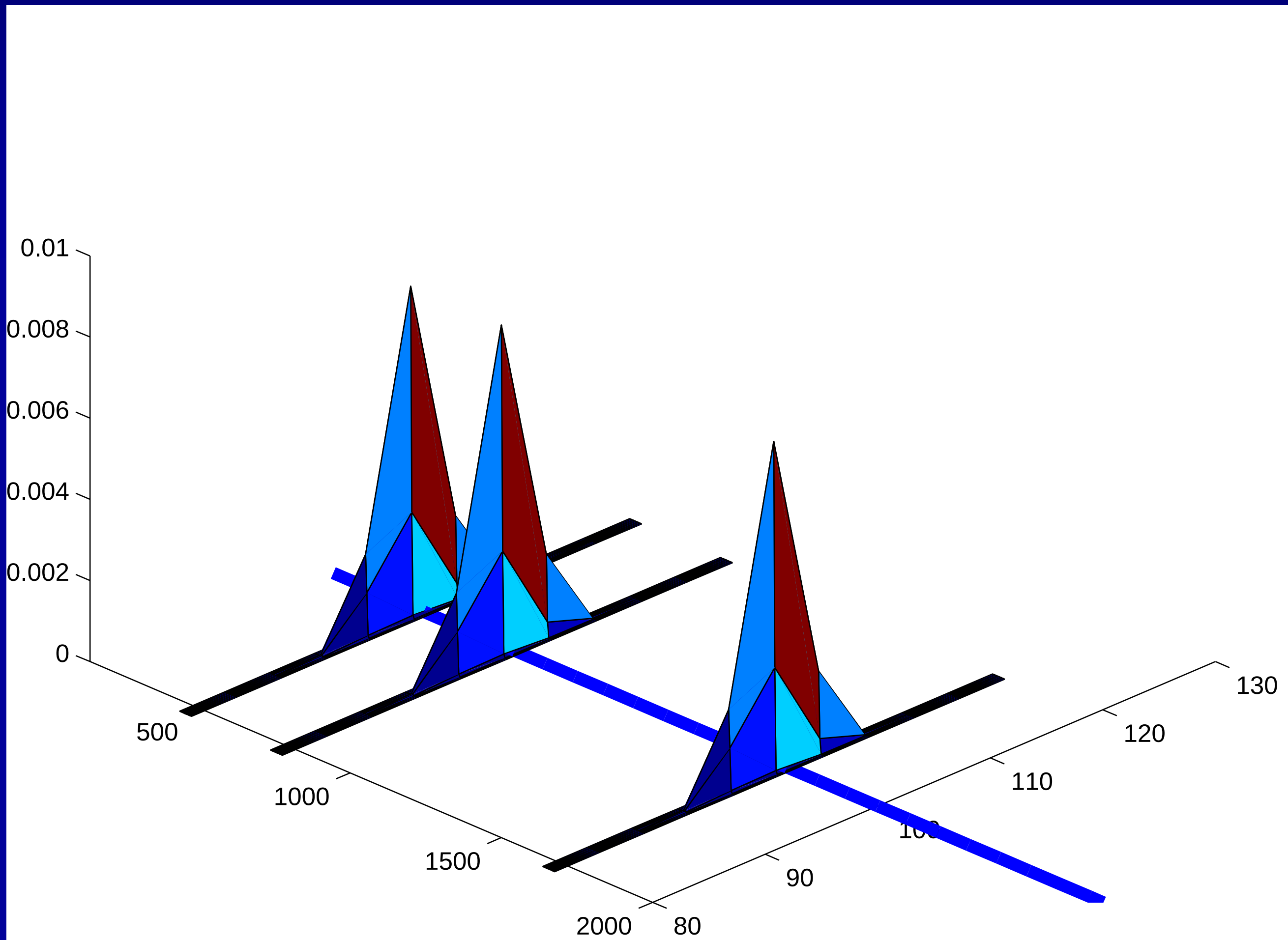
Type Identifiable Landmarks



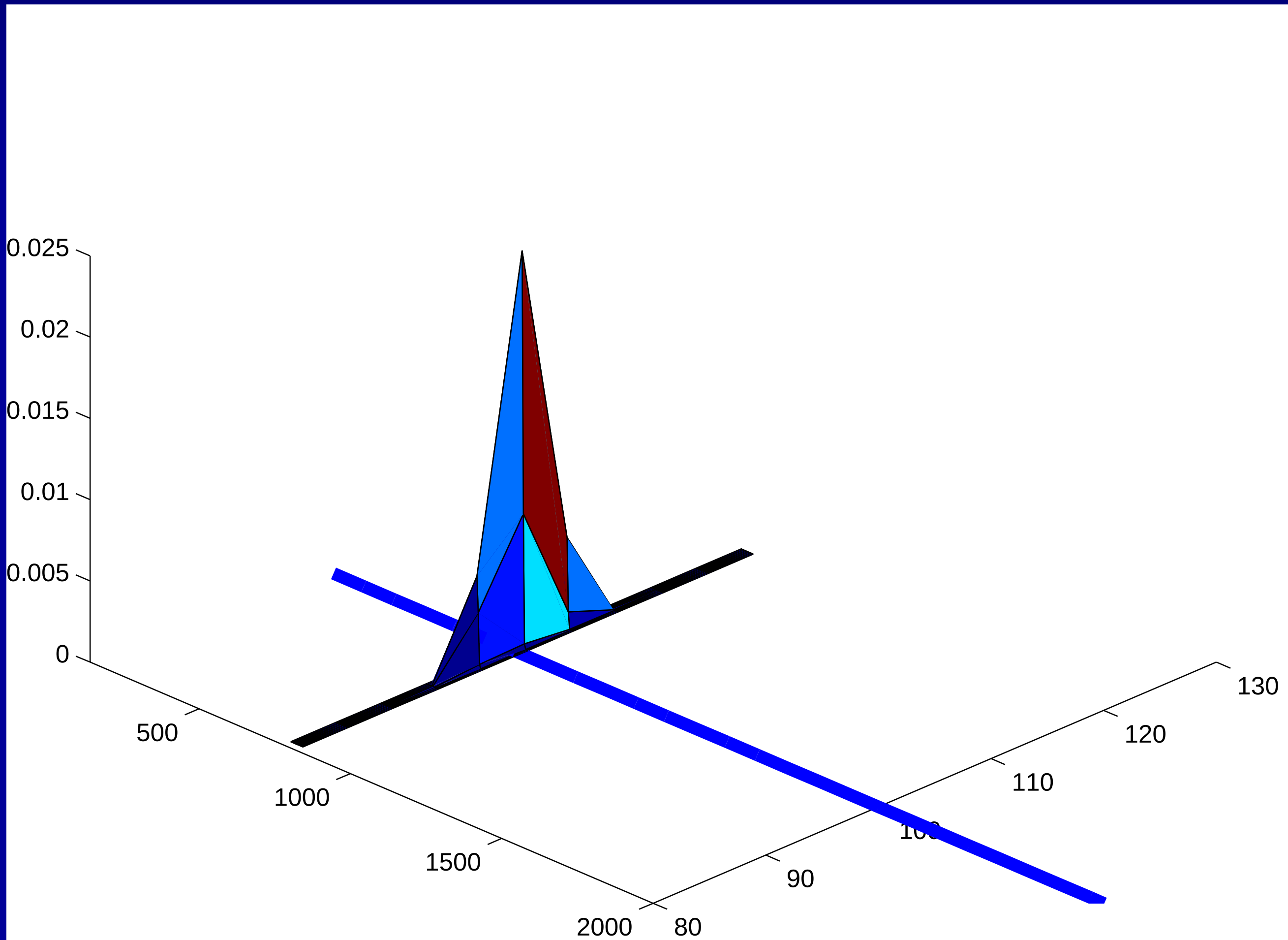
Type Identifiable Landmarks



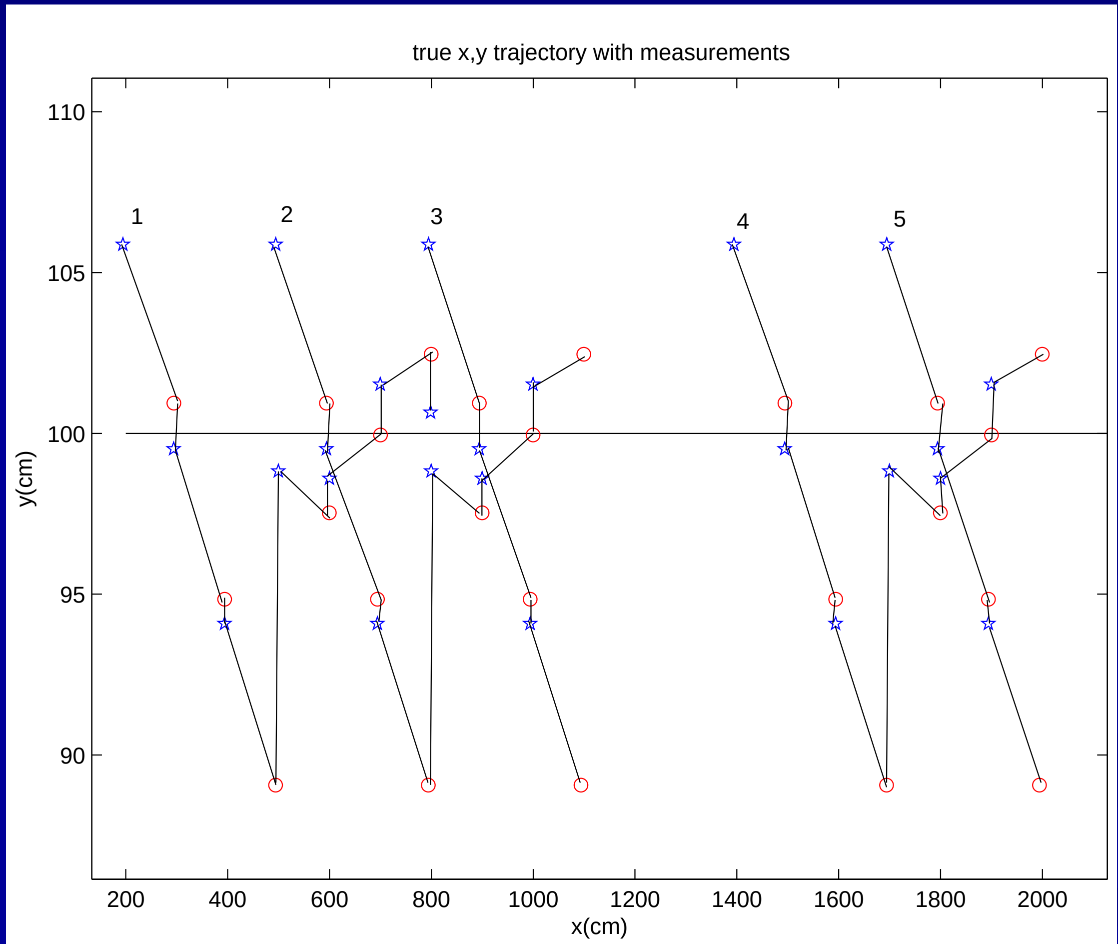
Type Identifiable Landmarks



Type Identifiable Landmarks



Type Identifiable Landmarks



Summary & Future

- Summary
  - Describing theory of localization and Kalman Filters
  - Illustrating applications of Kalman Filter to localization problems
  - Extension of Kalman Filter framework to multiple beliefs

- Future work
  - Practical application to robots
  - Possibilities of Kalman Filter extension

- Website: http://www.negenborn.net/kal_loc/

The end.